3

Exercício 3.1

```
f[x_, y_] = x^2 y;
```

a) $\partial_x f(0,0) = 0$

b) $\partial_x f(x_0, y_0) = 2\, x_0\, y_0$

c) $\partial_y f(1,2) = 1$

d) $\partial_y f(x_0, y_0) = x_0^2$

Exercício 3.2

a)

```
f[x_, y_] = 3 x^2 + 2 y^2;
```

$\partial_x f = 6\, x$

$\partial_y f = 4\, y$

b)

```
f[x_, y_] = Sin[x^2 - 3 x y];
```

$\partial_x f = (2\, x - 3\, y)\, \mathrm{Cos}[x^2 - 3\, x\, y]$

$\partial_y f = -3\, x\, \mathrm{Cos}[x^2 - 3\, x\, y]$

c)

```
f[x_, y_] = x^2 y^2 Exp[2 x y];
```

$\partial_x f = 2\, e^{2\, x\, y}\, x\, y^2 + 2\, e^{2\, x\, y}\, x^2\, y^3$

$\partial_y f = 2\, e^{2\, x\, y}\, x^2\, y + 2\, e^{2\, x\, y}\, x^3\, y^2$

d)

```
f[x_, y_] = Exp[x] Log[x y];
```

$$\partial_x f = \frac{e^x}{x} + e^x \,\text{Log}[x\,y]$$
$$\partial_y f = \frac{e^x}{y}$$

e)

```
f[x_, y_] = Exp[Sin[x √y]];
```

$$\partial_x f = e^{\text{Sin}\left[x\sqrt{y}\right]}\sqrt{y}\;\text{Cos}\left[x\sqrt{y}\right]$$
$$\partial_y f = \frac{e^{\text{Sin}\left[x\sqrt{y}\right]}\,x\,\text{Cos}\left[x\sqrt{y}\right]}{2\sqrt{y}}$$

f)

```
f[x_, y_] = (x^2 + y^2)/(x^2 - y^2);
```

$$\partial_x f = -\frac{4\,x\,y^2}{\left(x^2 - y^2\right)^2}$$
$$\partial_y f = \frac{4\,x^2\,y}{\left(x^2 - y^2\right)^2}$$

g)

```
f[x_, y_] = x Cos[x] Cos[y];
```

$\partial_x f =$ Cos[y] (Cos[x]-x Sin[x])
$\partial_y f =$ -x Cos[x] Sin[y]

h)

```
f[x_, y_] = ArcTan[x^2 y^3];
```

$$\partial_x f = \frac{2\,x\,y^3}{1 + x^4\,y^6}$$
$$\partial_y f = \frac{3\,x^2\,y^2}{1 + x^4\,y^6}$$

i)

```
f[x_, y_] = x + x y^2 + Log[Sin[x^2 + y]];
```

$\partial_x f = 1 + y^2 + 2\,x\,\mathrm{Cot}[x^2 + y]$

$\partial_y f = 2\,x\,y + \mathrm{Cot}[x^2 + y]$

j)

```
f[x_, y_, z_] = z Exp[x^2 + y^2];
```

$\partial_x f = 2\,e^{x^2+y^2}\,x\,z$

$\partial_y f = 2\,e^{x^2+y^2}\,y\,z$

$\partial_z f = e^{x^2+y^2}$

k)

```
f[x_, y_, z_] = Log[Exp[x] + z^y];
```

$\partial_x f = \dfrac{e^x}{e^x + z^y}$

$\partial_y f = \dfrac{z^y\,\mathrm{Log}[z]}{e^x + z^y}$

$\partial_z f = \dfrac{y\,z^{-1+y}}{e^x + z^y}$

l)

```
f[x_, y_, z_] = (x y^3 + Exp[z])/(x^3 y - Exp[z]);
```

$\partial_x f = -\dfrac{y\,\left(2\,x^3\,y^3 + e^z\,\left(3\,x^2 + y^2\right)\right)}{\left(e^z - x^3\,y\right)^2}$

$\partial_y f = \dfrac{2\,x^4\,y^3 - e^z\,x\,\left(x^2 + 3\,y^2\right)}{\left(e^z - x^3\,y\right)^2}$

$\partial_z f = \dfrac{e^z\,x\,y\,\left(x^2 + y^2\right)}{\left(e^z - x^3\,y\right)^2}$

Exercício 3.3

a)

$\partial_x f(0,0) =$ Não existe

$\partial_y f(0,0) =$ Não existe

b)

$\partial_x f(0,0) = 0$
$\partial_y f(0,0) = 0$

Exercício 3.4

a)

```
f[x_, y_] = Exp[x y];
```

$x\partial_x f = e^{x y}\, x\, y$
$y\partial_y f = e^{x y}\, x\, y$

b)

```
f[x_, y_] = Log[x^2 + y^2 + x y];
```

$$x\partial_x f = \frac{x\,(2\,x + y)}{x^2 + x\,y + y^2}$$
$$y\partial_y f = \frac{y\,(x + 2\,y)}{x^2 + x\,y + y^2}$$

$x\partial_x f + y\partial_y f = 2$

c)

```
f[x_, y_, z_] = x + (x - y)/(y - z);
```

$$\partial_x f = 1 + \frac{1}{y - z}$$
$$\partial_y f = \frac{-x + z}{(y - z)^2}$$
$$\partial_z f = \frac{x - y}{(y - z)^2}$$

$\partial_x f + \partial_y f + \partial_z f = 1$

Exercício 3.5

```
Ddirecao[f_, P_, u_] := Limit[(f @@ (P + h u) - f @@ P)/h, h → 0];
```

a)

```
f[x_, y_] = x^2 y + x; P = {1, 0}; u = {1, 1};
Ddirecao[f, P, Normalize[u]]
```

$\sqrt{2}$

b)

```
f[x_, y_] = x^2 Sin[2 y]; P = {1, π/2}; u = {3, -4};
Ddirecao[f, P, Normalize[u]]
```

$\frac{8}{5}$

c)

```
f[x_, y_, z_] = x^2 + y^2 + z^2; P = {1, 2, 3}; u = {1, 1, 1};
Ddirecao[f, P, Normalize[u]]
```

$4\sqrt{3}$

Exercício 3.6

a)

```
Grad[x Exp[-x + y], {x, y}]
```

$\{e^{-x+y} - e^{-x+y}\,x,\ e^{-x+y}\,x\}$

b)

```
Grad[r Sin[θ], {r, θ}]
```

{Sin[θ], r Cos[θ]}

c)

```
Grad[π r^2 h, {r, h}]
```

$\{2\,h\,\pi\,r,\ \pi\,r^2\}$

d)

```
Grad[x Exp[-x^2 - y^2 - z^2], {x, y, z}]
```

$$\left\{e^{-x^2-y^2-z^2} - 2\, e^{-x^2-y^2-z^2}\, x^2,\ -2\, e^{-x^2-y^2-z^2}\, x\, y,\ -2\, e^{-x^2-y^2-z^2}\, x\, z\right\}$$

e)

```
Grad[x y z / (x^2 + y^2 + z^2 + 1), {x, y, z}] // Simplify
```

$$\left\{\frac{y\, z\,(1 - x^2 + y^2 + z^2)}{(1 + x^2 + y^2 + z^2)^2},\ \frac{x\, z\,(1 + x^2 - y^2 + z^2)}{(1 + x^2 + y^2 + z^2)^2},\ \frac{x\, y\,(1 + x^2 + y^2 - z^2)}{(1 + x^2 + y^2 + z^2)^2}\right\}$$

f)

```
Grad[z^2 Exp[x] Cos[y], {x, y, z}] // Simplify
```

$$\{e^x z^2 \mathrm{Cos}[y],\ -e^x z^2 \mathrm{Sin}[y],\ 2\, e^x z\, \mathrm{Cos}[y]\}$$

Exercício 3.7

```
f[x_, y_] = x^2 + y^3;
```

Equação do plano tangente: $z = -11 + 6\,x + 3\,y$

Exercício 3.8

```
f[x_, y_] = x^2 + y^2;
```

Equação do plano tangente a f em (0,0): $z = 0$

```
g[x_, y_] = -x^2 - y^2 + x y^3;
```

Equação do plano tangente a g em (0,0): $z = 0$

Exercício 3.9

```
f[x_, y_] = Exp[x^2 - y^2];
```

a)

Equação do plano tangente a f em (1,1): $z = 1 + 2x - 2y$

Ponto de interseção do plano tangente com o eixo dos zz: $\{\{x \to 0, y \to 0, z \to 1\}\}$

b)

Cota do ponto do plano tangente correspondente a x=y=1: z=1.24

```
f[1, 1]
```

1

Exercício 3.10

Ver diapositivos

Exercício 3.11

Ver diapositivos

Exercício 3.12

a)

```
f[x_, y_] = x^2 y / (x^2 + y^2);
f[0, 0] = 0;
```

$$Df((0,0);(u1,u2)) = \frac{u1^2\, u2}{u1^2 + u2^2}$$

Se $(x,y) \neq (0,0)$ então $Df((x,y);(u1,u2)) = \dfrac{x\left(2\,u1\,y^3 + u2\left(x^3 - x\,y^2\right)\right)}{\left(x^2+y^2\right)^2}$

b)

f não é diferenciável na origem

Exercício 3.13

a)

```
f[x_, y_] = (x^2 + y^2) Sin[1/Sqrt[x^2 + y^2]];
f[0, 0] = 0;
```

$\partial_x f(0,0) = 0$

$\partial_y f(0,0) = 0$

b)

$$\partial_x f(x,y) = -\frac{x \operatorname{Cos}\left[\frac{1}{\sqrt{x^2+y^2}}\right]}{\sqrt{x^2+y^2}} + 2\,x \operatorname{Sin}\left[\frac{1}{\sqrt{x^2+y^2}}\right]$$

$$\partial_y f(x,y) = -\frac{y \operatorname{Cos}\left[\frac{1}{\sqrt{x^2+y^2}}\right]}{\sqrt{x^2+y^2}} + 2\,y \operatorname{Sin}\left[\frac{1}{\sqrt{x^2+y^2}}\right]$$

Created with the Wolfram Language