

→ Ficha 1 - Cálculo

1

1-

$$x^2 - 5x + 3 = 0$$

$$a^2 - 2ab + b^2$$

$$a^2 = x^2 \Rightarrow a = x$$

$$-2ab = -5x \Rightarrow b = \frac{5}{2}$$

$$\bullet x^2 - 5x + 3 = 0 \Rightarrow x^2 - 2x \cdot \left(\frac{5}{2}\right) + \left(\frac{5}{2}\right)^2 + 3 - \left(\frac{5}{2}\right)^2 = 0$$
$$\Rightarrow \underbrace{\left(x - \frac{5}{2}\right)^2}_{+3 - \frac{25}{4}} = 0$$

$$\Rightarrow \left(x - \frac{5}{2}\right)^2 - \frac{13}{4} = 0 \Rightarrow \left(x - \frac{5}{2}\right)^2 - \left(\frac{\sqrt{13}}{2}\right)^2 = 0$$

$$\Rightarrow \left(x - \frac{5}{2} - \frac{\sqrt{13}}{2}\right) \left(x - \frac{5}{2} + \frac{\sqrt{13}}{2}\right) = 0$$

$$\Rightarrow x - \frac{5 - \sqrt{13}}{2} = 0 \quad \vee \quad x - \frac{5 + \sqrt{13}}{2} = 0$$

$$\Rightarrow x = \frac{5 - \sqrt{13}}{2} \quad \vee \quad x = \frac{5 + \sqrt{13}}{2}$$

2-

$$t(x) = (m-2)x^2 - 2mx + 3m$$

=

→ Casos notáveis da multiplicação

$$\bullet a^2 - b^2 = (a-b)(a+b)$$

$$\bullet (a+b)^2 = a^2 + 2ab + b^2$$

$$\bullet (a-b)^2 = a^2 - 2ab + b^2$$

3-

$$a) p(x) = 2x^2 - 4x - 6$$

$$= 2 \left(\frac{x^2 - 2x - 3}{x^2 - 2ab} \right)$$

$$a^2 = x^2 \Rightarrow a = x$$

$$-2ab = -2x \Rightarrow b = 1$$

$$= 2 (x^2 - 2xz + 1 + 1 - 6 - 1)$$

$$= 2 ((x-1)^2 - 7)$$

$$= 2 ((x-1)^2 - (\sqrt{7})^2)$$

$$= 2 ((x-1-\sqrt{7})(x-1+\sqrt{7}))$$

$$b) \frac{x - 2\sqrt{x} - 8}{x^2 - 2ab} \quad x \geq 0$$

$$a^2 = x \Rightarrow a = \sqrt{x}$$

$$-2ab = -2\sqrt{x} \Rightarrow b = 1$$

$$= (\sqrt{x})^2 - 2\sqrt{x} + 1 + 1 - 8 - 1$$

$$= (\sqrt{x} - 1)^2 - 9$$

$$= (\sqrt{x} - 1)^2 - 3^2$$

$$= (\sqrt{x} - 1 - 3)(\sqrt{x} - 1 + 3)$$

$$= (\sqrt{x} - 4)(\sqrt{x} + 2)$$

4-

$$\begin{aligned}
 (a+b)^3 &= (a+b)^2(a+b) \\
 &= (a^2 + 2ab + b^2)(a+b) \\
 &= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3
 \end{aligned}$$

5-

$$a + (b-c) > b - 2a \Leftrightarrow$$

$$\Rightarrow a + 2a > b - b + c$$

$$\Rightarrow 3a > c$$

$$\Rightarrow a > \frac{c}{3} \quad \text{C.S.} =]\frac{c}{3}, +\infty[$$

6-

$$a < b$$

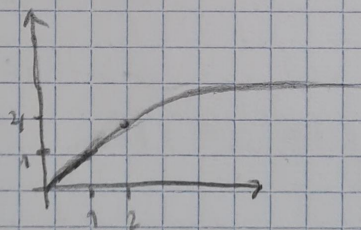
a) $|a| < |b|$

Falso, se $a = -1$ e $b = 0$

$$-1 < 0, \text{ mas } |-1| \not< |0|$$

b) $\sqrt{a} < \sqrt{b}$ ($a, b > 0$) Verdadeiro

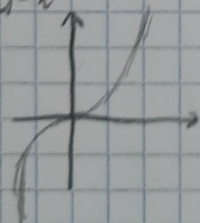
$$f(x) = \sqrt{x}$$

• A função f é crescente porque:

$$a < b \Rightarrow f(a) < f(b) \Rightarrow \sqrt{a} < \sqrt{b}$$

c) $a^3 < b^3$ verdadeiro

$$f(x) = x^3$$



Isso é, f é uma função crescente,
 $a < b \Rightarrow f(a) < f(b) \Rightarrow a^3 < b^3$

d) $\frac{1}{a} < \frac{1}{b}$

Falso, por ex: $a = 1$ e $b = 2$

$$1 < 2 \Rightarrow \frac{1}{1} > \frac{1}{2}$$

$$1 = 1 \not< 1 = 1$$

7-

$$a) \frac{(2 \times 3)^{10} + 2^9}{8^4} = \frac{(2^2 \times 3)^{10} + (2^2)^9}{(2^3)^4}$$

$$= \frac{3^{10} \times 2^{20} + 2^{18}}{2^{12}}$$

$$= \frac{3^{10} \times 2^{20}}{2^{12}} + \frac{2^{18}}{2^{12}}$$

$$= 3^{10} \times 2^8 + 2^6$$

$$= 2^6 (3^{10} \times 2^2 + 1)$$

$$= 2^3 (3^{10} \times 4 + 1)$$

$$c) x^{\frac{5}{2}} \left(x^{-\frac{3}{2}} + 2x^{\frac{1}{2}} + 3x^{\frac{7}{2}} \right) = \quad x > 0 \wedge x \neq 0$$

$$= x^1 + 2x^3 + 3x^5$$

d)

8-

$$\begin{array}{r|l}
 x^3 - 11x^2 + 37x - 30 & x^2 - 5x + 6 \\
 \underline{-x^3 + 5x^2 - 6x - 30} & x - 6 \\
 0 - 6x^2 + 31x - 30 & \\
 \underline{+6x^2 - 30x + 36} & \\
 0 + x + 6 &
 \end{array}$$

$$x^3 - 11x^2 + 37x - 30 = (x^2 - 5x + 6) \underbrace{(x - 6)}_{\text{quociente}} + \underbrace{(x + 6)}_{\text{resto}}$$

9-

a) $\{x \in \mathbb{R} : |x+4| = 3\}$

$$|x+4| = 3 \Rightarrow \begin{cases} x+4=3 \\ -x-4=3 \end{cases} \Rightarrow \begin{cases} x=-1 \\ x=-7 \end{cases}$$

b) $\{x \in \mathbb{R} : \sqrt{(x+1)^2} = 3\}$

$$\sqrt{(x+1)^2} = 3 \Rightarrow |x+1| = 3 \Rightarrow \begin{cases} x+1=3 \\ -x-1=3 \end{cases} \Rightarrow \begin{cases} x=2 \\ x=-4 \end{cases}$$

c) $\{x \in \mathbb{R} : |x| = |x+2|\}$

$$|x| = |x+2| \Rightarrow \begin{cases} x = x+2 \\ -x = x+2 \\ -x = x+2 \\ -x = -x-2 \end{cases} \Rightarrow \begin{cases} 2=0 \text{ Imp} \\ x=-1 \\ x=-1 \\ -2=0 \text{ Imp.} \end{cases}$$

$$d) \{x \in \mathbb{R} : (x^2 - 7)^2 = 0\}$$

$$x = \sqrt{7} \vee x = -\sqrt{7}$$

$$e) \{x \in \mathbb{R} : \sqrt{3x+1} = 2x\}$$

$$\sqrt{3x+1} = 2x \Rightarrow 3x+1 = 4x^2 \Rightarrow -4x^2 + 3x + 1 = 0$$

$$\Rightarrow x = \frac{-3 \pm \sqrt{(3)^2 - 4 \times (-4) \times 1}}{2 \times (-4)} \Rightarrow x = \frac{-3 \pm \sqrt{25}}{-8} \Rightarrow x = \frac{-3 \pm 5}{-8}$$

$$\Rightarrow x = 1 \vee x = -\frac{1}{4}$$

$$f) \{x \in \mathbb{R} : |x||2+3| = 4\}$$

$$|x||2+3| = 4 \Rightarrow \begin{cases} (x)(x+3) = 4 \\ (x)(-x-3) = 4 \\ (-x)(x+3) = 4 \\ (-x)(-x-3) = 4 \end{cases} \Rightarrow \begin{cases} x^2 + 3x - 4 = 0 \\ -x^2 - 3x - 4 = 0 \\ -x^2 - 3x - 4 = 0 \\ x^2 + 3x - 4 = 0 \end{cases} \Rightarrow \begin{cases} x = -4 \vee x = 1 \\ \text{Imp} \\ \text{Imp} \\ x = -4 \vee x = 1 \end{cases}$$

e. dwt

$$\bullet x^2 + 3x - 4 = 0 \Rightarrow x = \frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times (-4)}}{2 \times 1} \Rightarrow x = \frac{-3 \pm \sqrt{25}}{2}$$

$$\Rightarrow x = -4 \vee x = 1$$

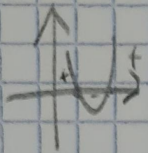
$$\bullet -x^2 - 3x - 4 = 0 \Rightarrow x = \frac{3 \pm \sqrt{(-3)^2 - 4 \times (-1) \times (-4)}}{2 \times (-1)} \Rightarrow x = \frac{3 \pm \sqrt{-7}}{-2} \text{ Imp.}$$

$$i) \{x \in \mathbb{R} : 6x^2 - 5x \leq -1\}$$

$$6x^2 - 5x + 1 \leq 0 \quad \text{e.s. } \left[\frac{1}{3}, \frac{1}{2}\right]$$

e. aus

$$6x^2 - 5x + 1 = 0 \Rightarrow x = \frac{+5 \pm \sqrt{(-5)^2 - 4 \cdot 6 \cdot 1}}{2 \cdot 6} \Rightarrow x = \frac{5 \pm \sqrt{1}}{12}$$



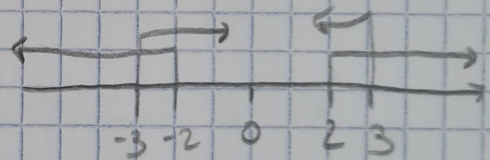
$$\Rightarrow x = \frac{1}{2} \vee x = \frac{1}{3}$$

$$j) \{x \in \mathbb{R} : |3x - 2| \leq 1 \quad \text{e.s. } \left[\frac{1}{3}, 1\right]$$

$$|3x - 2| \leq 1 \Rightarrow \begin{cases} 3x - 2 \leq 1 \\ -3x + 2 \leq 1 \end{cases} \Rightarrow \begin{cases} x \leq 1 \\ x \geq \frac{1}{3} \end{cases}$$

$$k) \{x \in \mathbb{R} : 2 < |x| < 3\} \quad]-3, -2[\cup]2, 3[$$

$$2 < |x| \Rightarrow \begin{cases} 2 < x \\ 2 < -x \end{cases} \Rightarrow \begin{cases} x > 2 \\ x < -2 \end{cases}$$



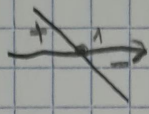
$$|x| < 3 \Rightarrow \begin{cases} x < 3 \\ x > -3 \end{cases}$$

$$l) \{x \in \mathbb{R} : |x - 1| < |x - 2|\}$$

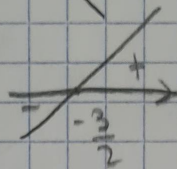
$$|x - 1| < |x - 2| \Rightarrow \begin{cases} x - 1 < x - 2 \\ x - 1 < -x + 2 \\ -x + 1 < x - 2 \\ -x + 1 < -x + 2 \end{cases} \Rightarrow \begin{cases} 0 < -1 \\ x < \frac{3}{2} \\ x > \frac{3}{2} \\ 0 < 1 \end{cases}$$

$$m) \{x \in \mathbb{R} : \frac{1-x}{2x+3} > 0\} \quad 2x+3 \neq 0 \Rightarrow x \neq -\frac{3}{2}$$

$$\bullet 1-x=0 \Rightarrow x=1$$



$$\bullet 2x+3=0 \Rightarrow x=-\frac{3}{2}$$



	$-\infty$	$-\frac{3}{2}$		1	$+\infty$
$1-x$	+	+	+	0	-
$2x+3$	-	0	+	+	+
$\frac{1-x}{2x+3}$	-	nd	+	0	-

$$e.s. =]-\frac{3}{2}, 1[$$

$$n) \{x \in \mathbb{R} : |x+2| + |x-2| < 10\}$$

$$e.s.]-5, 5[$$

$$|x+2| + |x-2| < 10 = \begin{cases} x+2 + x-2 < 10 \\ x+2 - x+2 < 10 \\ -x-2 + x-2 < 10 \\ -x-2 - x+2 < 10 \end{cases} \Rightarrow \begin{cases} x < 5 \\ 4 < 10 \\ -4 < 10 \\ x > -5 \end{cases}$$

$$o) \{x \in \mathbb{R} : |x^2-1| \leq 1\}$$

$$|x^2-1| \leq 1 \Rightarrow \begin{cases} x^2-1 \leq 1 \\ -x^2+1 \leq 1 \end{cases} \Rightarrow \begin{cases} x^2 \leq 2 \\ -x^2 \leq 0 \end{cases} \Rightarrow \begin{cases} \sqrt{x^2} \leq \sqrt{2} \\ x^2 \geq 0 \end{cases}$$

$$\Rightarrow \begin{cases} |x| \leq \sqrt{2} \\ |x| > 0 \text{ und alle reellen } x \end{cases} \Rightarrow \begin{cases} x \leq \sqrt{2} \vee x \geq -\sqrt{2} \end{cases} \quad e.s. = [-\sqrt{2}, \sqrt{2}]$$

$$p) \{x \in \mathbb{R} : 2x^2 \leq 4\} \quad e.s. [-\sqrt{2}, \sqrt{2}]$$

$$2x^2 \leq 4 \Rightarrow x^2 \leq 2 \Rightarrow \sqrt{x^2} \leq \sqrt{2} \Rightarrow |x| \leq \sqrt{2} \Rightarrow \begin{cases} x \leq \sqrt{2} \\ x \geq -\sqrt{2} \end{cases}$$

10-

$$|x-a| < \varepsilon$$

$$a)]-2, 2[\quad |x| < 2 \Rightarrow \begin{cases} x < 2 \\ -x < 2 \Rightarrow x > -2 \end{cases}$$

$$b)]-4, 0[\quad |x+2| < 2 \Rightarrow \begin{cases} x+2 < 2 \\ -x-2 < 2 \end{cases} \Rightarrow \begin{cases} x < 0 \\ x > -4 \end{cases}$$

$$c)]0, 4[\quad |x-2| < 2 \Rightarrow \begin{cases} x-2 < 2 \\ -x+2 < 2 \end{cases} \Rightarrow \begin{cases} x < 4 \\ x > 0 \end{cases}$$

$$d)]-3, 7[\quad |x-2| < 5 \Rightarrow \begin{cases} x-2 < 5 \\ -x+2 < 5 \end{cases} \Rightarrow \begin{cases} x < 7 \\ x > -3 \end{cases}$$

11-

$$a) \{x \in \mathbb{R} : 1-x \leq 2\}$$

$$1-x \leq 2 \Rightarrow x \geq -1$$

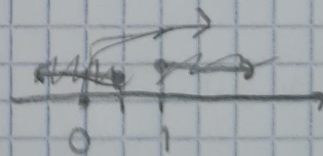
$$[-1, +\infty[$$

$$b) \{x \in \mathbb{R} : 0 \leq 1-2x \leq -1\}$$

$$]-\infty, \frac{1}{2}] \cup [1, +\infty[$$

$$\bullet 0 \leq 1-2x \Rightarrow \frac{1}{2} \geq x \Rightarrow x \leq \frac{1}{2}$$

$$\bullet 1-2x \leq -1 \Rightarrow x \geq 1$$



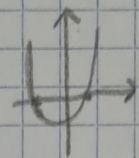
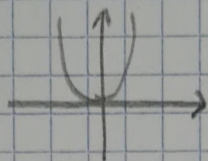
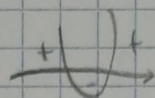
$$c) \{x \in \mathbb{R} : x^2 > 5\} \quad]-\infty, -\sqrt{5}[\cup]\sqrt{5}, +\infty[$$

$$x^2 > 5 \Rightarrow \sqrt{x^2} > \sqrt{5} \Rightarrow |x| > \sqrt{5} \Rightarrow \begin{cases} x > \sqrt{5} \\ x < -\sqrt{5} \end{cases}$$

$$d) \{x \in \mathbb{R} : x^2(x^2 - 1) \geq 0\}$$

$$e.s. =]-\infty, -1] \cup [1, +\infty[$$

$$x^2(x^2 - 1) \geq 0$$



e. aux

$$x^2 = 0 \Rightarrow x = 0$$

$$x^2 - 1 = 0 \Rightarrow x = 1 \vee x = -1$$

	$-\infty$	-1	0	1	$+\infty$
x^2	+	+	0	+	+
$x^2 - 1$	+	0	-	0	+
$x^2(x^2 - 1)$	+	0	-	0	+

$$e) \{x \in \mathbb{R} : |5 - \frac{1}{x}| < 1, x \neq 0\}$$

$$|5 - \frac{1}{x}| < 1 \Rightarrow \begin{cases} 5 - \frac{1}{x} < 1 \\ -5 + \frac{1}{x} < 1 \end{cases} \Rightarrow \begin{cases} \frac{1}{x} > 4 \\ \frac{1}{x} < 6 \end{cases} \Rightarrow \begin{cases} x < \frac{1}{4} \\ x > \frac{1}{6} \end{cases}$$

$$e.s. =]\frac{1}{6}, \frac{1}{4}[$$

$$f) \{x \in \mathbb{R} : |3 - x| \geq 2\}$$

$$e.s. =]-\infty, 1] \cup [5, +\infty[$$

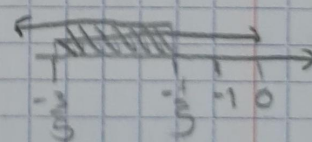
$$|3 - x| \geq 2 \Rightarrow \begin{cases} 3 - x \geq 2 \\ -3 + x \geq 2 \end{cases} \Rightarrow \begin{cases} x \leq 1 \\ x \geq 5 \end{cases}$$

$$g) \{x \in \mathbb{R} : |5x + 2| \leq 1\}$$

$$[-\frac{3}{5}, -\frac{1}{5}]$$

$$|5x + 2| \leq 1 \Rightarrow \begin{cases} 5x + 2 \leq 1 \\ -5x - 2 \leq 1 \end{cases}$$

$$\Rightarrow \begin{cases} x \leq -\frac{1}{5} \\ x \geq -\frac{3}{5} \end{cases}$$



$$h) \{x \in \mathbb{R} : x^3 \geq 4x\}$$

$$e.s. = [-2, 0] \cup [2, +\infty[$$

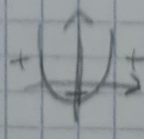
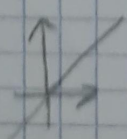
$$x^3 - 4x \geq 0$$

$$\Rightarrow x = 0 \vee x = 2 \vee x = -2$$

e. aux

$$x^3 - 4x = 0 \Rightarrow$$

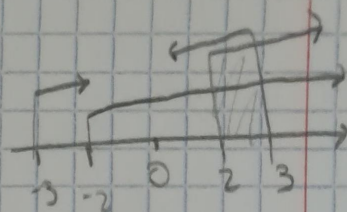
$$\Rightarrow x(x^2 - 4) = 0$$



	$-\infty$	-2	0	2	$+\infty$
x	-	-	0	+	+
$x^3 - 4$	+	0	-	0	+
$x^3 - 4x$	-	0	+	0	+

$$9) \{x \in \mathbb{R} : 4 < x^2 < 9\}$$

$$\bullet 4 < x^2 \Rightarrow x^2 > 4 \Rightarrow |x| > 2 \Rightarrow \begin{cases} x > 2 \\ x < -2 \end{cases}$$



$$\bullet x^2 < 9 \Rightarrow |x| < 3 \Rightarrow \begin{cases} x < 3 \\ x > -3 \end{cases}$$

$$12) \{x \in \mathbb{R} : \frac{x}{x-2} \leq 0\} \wedge x \neq 2$$

$$\bullet x-2=0 \Rightarrow x=2 \quad \text{---} \rightarrow$$

	$-\infty$	0	2	$+\infty$
x	-	0	+	+
$x-2$	-	-	0	+
$\frac{x}{x-2}$	+	0	-	+

O.S. $[0, 2[$

12-

$a=b$ Entw

$$a^2 = ab \Rightarrow a^2 - b^2 = ab - b^2$$

$$\Rightarrow (a-b)(a+b) = b(a-b)$$

Se $a=b$ Entw

$$a-b=0$$

$$\Rightarrow a+b=b \Rightarrow 2b=b \Rightarrow 2=1$$

es ist a pool
dividen von 0

$\rightarrow$ • forma contraria

$$(a-b)(a+b) = b(a-b)$$

$$\Rightarrow (a-b)(a+b) - b(a-b) = 0$$

$$\Rightarrow (a-b)[(a+b) - b] = 0$$

$$\Rightarrow a-b=0 \vee a+b-b=0$$

$$\Rightarrow a=b \vee a=0$$

13-

a) $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$ Falso, $\sqrt{25+9} \neq \sqrt{25} + \sqrt{9}$
 $\sqrt{36} \neq 5 + 3$
 $6 \neq 8$

b) $\sqrt{xy} = \sqrt{x}\sqrt{y}$ Falso $x=1$ $y=-1 \Rightarrow \sqrt{1 \times (-1)} \neq \sqrt{1} \times \sqrt{-1}$
 $\sqrt{-1}$ no está definido

c) $(x+y)^n = x^n + y^n$ Falso porque $(2+3)^2 \neq 2^2 + 3^2$
 $25 \neq 13$

d) $(xy)^n = x^n y^n$ Verdadero

e) $\frac{1}{x+y} = \frac{1}{x} + \frac{1}{y}$ Falso $\frac{1}{2+3} \neq \frac{1}{2} + \frac{1}{3}$
 $\frac{1}{5} \neq \frac{5}{6}$

f) $|x+y| = |x| + |y|$ Falso $|-2+3| \neq |-2| + |3|$
 $\Rightarrow |1| \neq 5 \Rightarrow 1 \neq 5$