→ Calculo - Ficha 2

1-

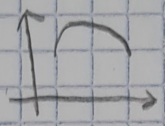
a) $y = -(x+1)^2$ $\quad D = \mathbb{R}$ $\quad CD =]-\infty, 0]$

b) $x - y = \frac{1}{x} \Rightarrow y = x - \frac{1}{x}$ $\quad D = \mathbb{R} \setminus \{0\}$

c) $(x-1)^2 + (y-1)^2 = 1 \Rightarrow y - 1 = \pm\sqrt{1-(x-1)^2}$

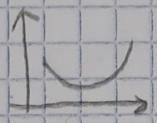
$\Rightarrow y = 1 \pm \sqrt{1-(x-1)^2}$

- $y = 1 + \sqrt{1-(x-1)^2}$



$D = [0, 2]$

- $y = 1 - \sqrt{1-(x-1)^2}$



d) $y(x^2+1) = x^2 - 1 \Rightarrow y = \frac{x^2-1}{(x^2+1)}$ $\quad D = \mathbb{R}$

2-

a) $y = \frac{2x^2-3}{7x-4}$ $\Rightarrow 2$

- tem 2 zeros
- é uma função racional

$Df = \{x \in \mathbb{R} : 7x - 4 \neq 0\} = \mathbb{R} \setminus \{\frac{4}{7}\}$

$2x^2 - 3 = 0 \Rightarrow x = \pm\sqrt{\frac{3}{2}} \Rightarrow x = \pm\frac{\sqrt{6}}{2}$

b) $y = \frac{3x^2-8x-3}{3x^2+2}$ $\Rightarrow 3$

- tem dois zeros
- o domínio é $\mathbb{R}$ logo está sempre definida

$Df = \{x \in \mathbb{R} : 3x^2 + 2 \neq 0\} = \mathbb{R}$

$5x^2 - 8x - 3 = 0 \Rightarrow x = \frac{+8 \pm \sqrt{(-8)^2 - 4 \times 5 \times (-3)}}{2 \times 5}$

$\Rightarrow x = \frac{8 \pm \sqrt{64+60}}{10} \Rightarrow x = \frac{8 \pm 2\sqrt{31}}{10} \Rightarrow x = \frac{4 \pm \sqrt{31}}{5}$

e) $y = \frac{11x+2}{2x^3-1} \Rightarrow 1$

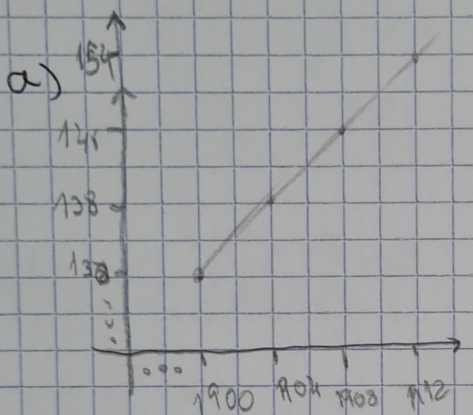
$D = \{x \in \mathbb{R} : \underbrace{2x^3 - 1 = 0}_{x = \sqrt[3]{\frac{1}{2}}}\} = \mathbb{R} \setminus \{\sqrt[3]{\frac{1}{2}}\}$

- tem 1 zero
- não está definido em $\sqrt[3]{\frac{1}{2}}$

$11x + 2 = 0 \Rightarrow x = -\frac{2}{11}$

3-

a)



b)

$D_a = \{1900, 1904, 1908, 1912\}$

$CD_a = \{130, 138, 146, 154\}$

c)

$2020 - 1912 = 108$

$108 : 4 = 27$

$a(2020) = a(1912) + 8 \times 27$

$= 370$

4.

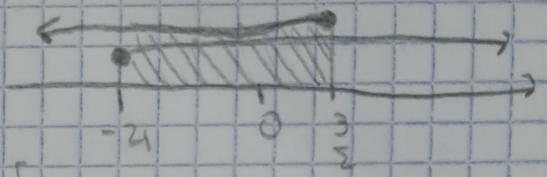
a) $f(x) = \sqrt{x+5}$ $\quad g(x) = \sqrt{x+5}$ $\quad D_{f+g} =]-5, +\infty[$

$D_f = D_g = \{x \in \mathbb{R} : \underbrace{x + 5 \geq 0}_{x \geq -5}\} = [-5, +\infty[$

$D_{f+g} = D_f \cap D_g = [-5, +\infty[= D_f = D_g = D_{fg}$

b) $f(x) = \sqrt{3-2x}$ $\quad g(x) = \sqrt{x+4}$

$D_f = \{ x \in \mathbb{R} : \underbrace{3-2x \geq 0}_{x \leq \frac{3}{2}} \} =]-\infty, \frac{3}{2}]$



$D_g = \{ x \in \mathbb{R} : \underbrace{x+4 \geq 0}_{x \geq -4} \} = [-4, +\infty[$

$D_{f+g} = D_{f-g} = D_{fg} = [-4, \frac{3}{2}]$ $\quad D_{f/g} = [-4, \frac{3}{2}[$

5 -

a) $f(x) = x^2 - 3x$ $\quad g(x) = \sqrt{x+2}$

$(f \circ g)(x) = f(g(x))$ $\quad D_f = \mathbb{R}$

$D_g = \{ x \in \mathbb{R} : \underbrace{x+2 \geq 0}_{x \geq -2} \} = [-2, +\infty[$

1º Domínio

$D_{f \circ g} = \{ x \in D_g : g(x) \in D_f \}$

$= \{ x \in [-2, +\infty[: \sqrt{x+2} \in \mathbb{R} \}$

e. duv

$x \in [-2, +\infty[\Rightarrow x \geq -2$

$x \in \mathbb{R} \Rightarrow -\infty < \sqrt{x+2} < +\infty$

$\Rightarrow x \geq -2 \wedge \sqrt{x+2} < +\infty$

$\Rightarrow x \in [-2, +\infty[$

Assim

$f \circ g \; [-2, +\infty[\longrightarrow \mathbb{R}$

$x \longmapsto x+2-3\sqrt{x+2}$

2º Lei

$f \circ g(x) = f(\sqrt{x+2}) = (\sqrt{x+2})^2 - 3\sqrt{x+2} = x+2-3\sqrt{x+2}$

$D_f = \mathbb{R}$

$D_g = [-2, +\infty[$

- $f \circ f$

1º Domínio

$$D_{f \circ f} = \{ x \in D_f : f(x) \in D_f \}$$

$$= \{ x \in \mathbb{R} : x^2 - 3x \in \mathbb{R} \} = \mathbb{R}$$

② lei

$$(f \circ f)(x) = f(f(x)) = (x^2 - 3x)^2 - 3(x^2 - 3x)$$

$$= x^4 - 6x^3 + 9x^2 - 3x^2 + 9x$$

$$= x^4 + 9x$$

Anim

$f \circ f \; \mathbb{R} \to \mathbb{R}$

$x \mapsto x^4 + 9x$

- $g \circ f$

1º Domínio

$$D_{g \circ f} = \{ x \in D_f : f(x) \in D_g \}$$

$$= \{ x \in \mathbb{R} : x^2 - 3x \in [-2, +\infty[\}$$

c.aux

$x \in \mathbb{R}$

$x^2 - 3x \in [-2, +\infty[$

$\Rightarrow x^2 - 3x \geq -2 \Rightarrow x^2 - 3x + 2 \geq 0$

$\Rightarrow -\infty \leq x \leq 1 \wedge 2 \leq x \leq +\infty$

c.aux

$x^2 - 3x + 2 = 0 \Leftrightarrow$

$\Rightarrow x = \frac{+3 \pm \sqrt{(-3)^2 - 4 \times 1 \times 2}}{2 \times 1}$

$\Leftrightarrow x = 2 \vee x = 1$

| | − | | + |

Lei $(g \circ f)(x) = \sqrt{x^2 - 3x + 2}$

Anim

$]-\infty, 1] \cup [2, +\infty[\to \mathbb{R}$

$x \longmapsto \sqrt{x^2 - 3x + 2}$

5-

a)

• $g \circ g$ $\qquad D_g = [-2, +\infty[$

1º Domínio

$D_{g \circ g} = \{ x \in D_g : g(x) \in D_g \}$

$= \{ x \in [-2, +\infty[: \sqrt{x+2} \in [-2, +\infty[\}$

• $x \in [-2, +\infty[\Rightarrow x \geq -2$

• $\sqrt{x+2} \in [-2, +\infty[\Rightarrow \sqrt{x+2} \geq -2$

$\Rightarrow \sqrt{x+2} \geq -2 \wedge x \geq -2$

$\Rightarrow x + 2 \geq 4 \wedge x \geq -2$

$\Rightarrow x \geq 2 \wedge x \geq -2$

$\Rightarrow x \in [2, +\infty[$

Assim

$g \circ g \quad [2, +\infty[\to \mathbb{R}_0^+$

$x \longmapsto \sqrt{\sqrt{x+2}+2}$

② Lei

$(g \circ g)(x) = g(\sqrt{x+2}) = \sqrt{\sqrt{x+2}+2}$

b) $f(x) = \sqrt{x+15} \qquad g(x) = x^2 + 2x$

$f \circ f(x)$ $\qquad D_f = [-15, +\infty[$

$D_{f \circ f} = \{ x \in D_f : f(x) \in D_f \}$

$= \{ x \in [-15, +\infty[: \sqrt{x+15} \in [-15, +\infty[\}$

• $x \geq -15 \quad \wedge \quad \sqrt{x+15} \geq -15$

$\Rightarrow x \geq -15 \quad \wedge \quad x \geq 210$

Assim

$f \circ f \quad [210, +\infty[\to \mathbb{R}^+$

$x \to \sqrt{\sqrt{x+15}+15}$

• $f \circ g$

$D_f = [-15, +\infty[$

$D_g = \mathbb{R}$

1° Dominio

$D_{f \circ g} = \{x \in D_g : g(x) \in D_f\}$

$= \{x \in \mathbb{R} : x^2 + 2x \in [-15, +\infty[\}$

• $x \in \mathbb{R} \wedge x^2 + 2x \geq -15$

$x^2 + 2x + 15 \geq 0$

$\wedge \quad x \in \mathbb{R}$

Eq. aux

$x^2 + 2x + 15 = 0$

$\Rightarrow x = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times 15}}{2 \times 1}$

$\Rightarrow x < \frac{-2 \pm \sqrt{-56}}{2}$ Impossível

$+ \quad (+) \quad +$

Assim:

$f \circ g \quad \mathbb{R} \to \mathbb{R}^+$

$x \mapsto \sqrt{x^2 + 2x + 15}$

2 Formula

$f \circ g(x) = f(g(x)) = \sqrt{x^2 + 2x + 15}$

• $g \circ f$

$D_{g \circ f} = \{x \in D_f : f(x) \in D_g\}$

$= \{x \in [-15, +\infty[: \sqrt{x + 15} \in \mathbb{R}$

• $x \geq -15 \wedge \sqrt{x + 15} \in \mathbb{R}$

5-

b) $g \circ g$

$D_{g \circ g} = \{ x \in D_g : g(x) \in D_g \}$

$= \{ x \in \mathbb{R} : x^2 + 2x \in \mathbb{R} \}$

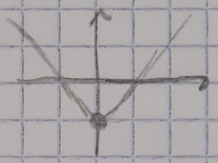
Formula

$g \circ g(x) = g(g(x)) = (x^2 + 2x)^2 + 2(x^2 + 2x)$

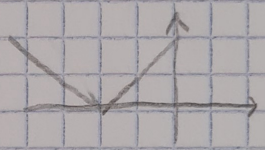
$= x^4 + 4x^3 + 4x^2$

6-

a) $g(x) = f(x) - 1 = |x| - 1$



b) $g(x) = f(x+2) = |x+2|$



c) $g(x) = \max \{ f(x), 1 \}$

- $g(-1) = \max \{ |-1|, 1 \} = 1$
- $g(0) = \max \{ |0|, 1 \} = 1$

$g(1) = \max \{ |1|, 1 \} = 1$

- Função teto $D = \mathbb{R}$

$\lceil x \rceil = \min \{ m \in \mathbb{Z} : m \geq x \}$

menor dos maiores imediatamente a seguir a x

- Função chão

$\lfloor x \rfloor = \max \{ m \in \mathbb{Z} : m \leq x \}$

o maior dos que são menores que x

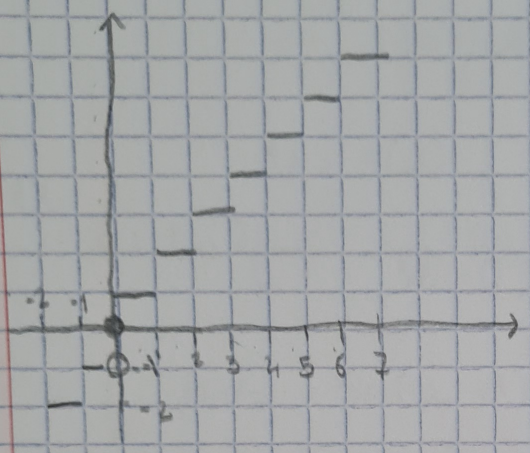
7-

a) $D_m = \{ 1,2 ; 2,1 ; 4,6 ; 5,3 ; 6,8 \}$ $\quad f(x) = \lceil x \rceil$

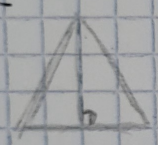
x	1,2	2,1	4,6	5,3	6,8
$f(x)$	2	3	5	6	7

b) $D = [-2, 7]$

$$g(x) = \begin{cases} \lceil x \rceil & x \geq 0 \\ \lfloor x \rfloor & x < 0 \end{cases}$$



8-



$$A = \frac{l \times \frac{l\sqrt{3}}{2}}{2} = \frac{l^2\sqrt{3}}{4} \qquad P = 3l$$

$$l^2 = \left(\frac{l}{2}\right)^2 + x^2 \Rightarrow l^2 - \frac{l^2}{4} = x^2$$

$$\Rightarrow \frac{3l^2}{4} = x^2 = \frac{l\sqrt{3}}{2}$$

9-

$P(x, y)$

$2x + 4y = 5 \Rightarrow 4y = 5 - 2x \Rightarrow y = -\frac{1}{2}x + \frac{5}{4}$

$$d(PO) = \sqrt{(x-0)^2 + \left(y - \left(-\frac{1}{2}x + \frac{5}{4}\right)\right)^2}$$

10-

a) $f(x) = x$ $\quad D = \mathbb{R}$ $\quad CD = \mathbb{R}$ $\quad y = x$

b) $f(x) = x^2$ $\quad D = [0, +\infty[$ $\quad CD = \mathbb{R}_0^+$ $\quad f(x) = \sqrt{x}$

c) $f(x) = x - 3$ $\quad D = \mathbb{R}$ $\quad CD = \mathbb{R}$

$y = x - 3 \Rightarrow y + 3 = x$ $\quad f^{-1}(x) = x + 3$

d) $f(x) = x^3$ $\quad D = [0, +\infty[$ $\quad CD = \mathbb{R}_0^+$

$f(x)^{-1} = \sqrt[3]{x}$

e) $f(x) = \sqrt{x+2}$ $\quad D = [-2, +\infty[$ $\quad CD = \mathbb{R}_0^+$

$y = \sqrt{x+2} \Rightarrow x = y^2 - 2$ $\quad f(x) = x^2 - 2$

f) $f(x) = \frac{1}{x^2+5}$ $\quad D = \mathbb{R}$ $\quad CD =]0, 1]$

$y = \frac{1}{x^2+5} \Rightarrow \frac{1}{y} = x^2 + 5 \Rightarrow x = \pm\sqrt{\frac{1}{y} - 5}$

g)

12-

$f(x) = \cos x$

$g(x) = x^2$

$(f \circ g)(x) = f(g(x)) = \cos x^2$

$(g \circ f)(x) = g(f(x)) = (\cos x)^2 = \cos^2 x$

$(f \circ f)(x) = f(f(x)) = \cos(\cos x)$

13-

a) $\frac{\cos 2x + 1}{2} = \frac{1}{2}(\cos 2x + 1)$

$\cos^2 x + \operatorname{sen}^2 x = 1$

$\cos(2x) = \cos^2 x - \operatorname{sen}^2 x$

$= 2\cos^2 x - 1$

$= \frac{1}{2}(\cos^2 x - \operatorname{sen}^2 x + 1)$

$= \frac{1}{2}[\cos^2 + (1 - \cos^2 x) + 1) = \frac{1}{2}(2\cos^2 x) = \cos^2 x$ e.q.m

b) $\frac{1 - \cos 2x}{2} = \frac{1}{2}(1 - \cos 2x)$

$= \frac{1}{2}(1 - (\cos^2 x - \operatorname{sen}^2 x)) = \frac{1}{2}(1 - (1 - \operatorname{sen}^2 x - \operatorname{sen}^2 x))$

$= \frac{1}{2}(2\operatorname{sen}^2 x) = \operatorname{sen}^2 x$ e.q.m

$\cos(\alpha + \beta) = \cos\alpha + \cos\beta - \operatorname{sen}\alpha \operatorname{sen}\beta$

$\operatorname{sen}(2x) = 2\operatorname{sen}x \cos x$

c) $\cos(3x) = \cos(x + 2x)$

$= \cos x \cos(2x) + \operatorname{sen}x \operatorname{sen}(2x)$

$= \cos x (2\cos^2 x - 1) - \operatorname{sen}x (2 \operatorname{sen}x \cos x)$

$= 2\cos^3 x - \cos x - 2\operatorname{sen}^2 x \cos x$

$= 2\cos^3 x - \cos x - (1 - \cos^2 x) 2\cos x$

$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$

$= 4\cos^3 x - 3\cos x$ e.q.m

14 -

a) $\operatorname{sen}(2x) = \frac{1}{2} \Rightarrow$

$\Rightarrow 2x = \frac{\pi}{6} + 2k\pi \quad \vee \quad 2x = \pi - \frac{\pi}{6} + 2k\pi \quad , k \in \mathbb{Z}$

$\Rightarrow x = \frac{\pi}{12} + k\pi \quad \vee \quad x = \frac{5\pi}{12} + k\pi, \quad k \in \mathbb{Z}$

b) $\sqrt{3}\operatorname{sen}(3x) + \cos 3x = 2$

$\Rightarrow \sqrt{3}\operatorname{sen}(2x+x) + \cos(2x+x) = 2$

$\Rightarrow \sqrt{3}\operatorname{sen}2x\cos x + \cos 2x \operatorname{sen} x + \cos 2x \cos x - \operatorname{sen} 2x \operatorname{sen} x = 2$

$\Rightarrow \sqrt{3}2\operatorname{sen} x \cos x \cos x + \cos^2 x - \operatorname{sen}^2 x + \cos^2 x - \operatorname{sen}^2 x - 2\operatorname{sen} x \cos x \operatorname{sen} x = 2$

$\Rightarrow 2\sqrt{3}\operatorname{sen} x \cos^2 x + 2\cos^2 x - 2\operatorname{sen}^2 x - 2\operatorname{sen}^2 x \cos x = 2$

$\Rightarrow \sqrt{3}\operatorname{sen} x \cos^2 x + \cos^2 x - \operatorname{sen}^2 x - \operatorname{sen}^2 x \cos x = 1$

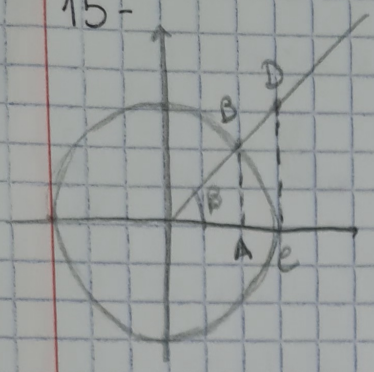
$\Rightarrow \sqrt{3}\operatorname{sen} x (1 - \operatorname{sen}^2 x) + (1 - \operatorname{sen}^2 x) - \operatorname{sen}^2 x - \operatorname{sen}^2 x \cos x = 1$

$\Rightarrow \sqrt{3}\operatorname{sen} x - \operatorname{sen}^3 x + 1 - \operatorname{sen}^2 x - \operatorname{sen}^2 x - \operatorname{sen}^2 \cos x = 1$

c) $4\cos^3 x - 3\cos x = \frac{1}{2}$

$\Rightarrow \cos(3x) = \frac{1}{2} \Leftrightarrow 3x = \frac{\pi}{3} + 2k\pi \quad \vee \quad 3x =$

15-



$\triangle OCB$ $\quad area_1 = \frac{\overline{OC} \times \overline{AB}}{2} = \frac{1 \times \operatorname{sen} B}{2}$

arco $\nwarrow \measuredangle BOC$ $\quad area_2 = \frac{1}{2} B \times \underbrace{2^2}_{\hookrightarrow = 1}$

$\triangle OCD$ $\quad area_3 = \frac{\overline{OC} \times \overline{CD}}{2} = \frac{1}{2} \operatorname{tg} B$

Temos:

$area_1 < area_2 < area_3$

$\frac{1}{2} \operatorname{tg} B < \frac{1}{2} B < \frac{1}{2} \operatorname{sen} B$ $\quad \times \frac{1}{\operatorname{sen} B}$

$\Rightarrow 1 < \frac{B}{\operatorname{sen} B} < \frac{1}{\cos B}$

$\Rightarrow 1 > \frac{\operatorname{sen} B}{B} > \cos B \Rightarrow \cos B < \frac{\operatorname{sen} B}{B} < B$

17-

a) $\operatorname{arcsen}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

b) $\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

c) $\operatorname{arctg}(-1) = \frac{3\pi}{4}$

d) $\operatorname{sen}\left(\operatorname{arcsen}\left(-\frac{1}{2}\right)\right) = -\frac{1}{2}$

e) $\operatorname{arcsen}\left(\operatorname{sen}\frac{7\pi}{6}\right) = \operatorname{arcsen}\left(\operatorname{sen}\left(-\frac{\pi}{6}\right)\right) = -\frac{\pi}{6}$

f) $\cos\left(\arccos\left(\frac{\sqrt{3}}{2}\right)\right) = \frac{\sqrt{3}}{2}$

g) $\sec\left(\arccos\frac{1}{2}\right) = \frac{1}{\cos\left(\arccos\frac{1}{2}\right)} = \frac{1}{\frac{1}{2}} = 2$

h) $\operatorname{tg}\left(\operatorname{arcsen}\left(-\frac{1}{2}\right)\right) = \operatorname{tg}\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{3}$

$\frac{1}{2}$

$\frac{\sqrt{3}}{2}$

18-

a) $h(x) = \operatorname{sen}(\arccos x) = \sqrt{1-x^2}$

Notemos que: $h(x) = \operatorname{sen}(\arccos x)$

$= \operatorname{sen} x \circ \arccos x$

função composta

1) Domínio

$D = \{ x \in D_{\arccos} : \arccos x \in D_{\operatorname{sen}} \}$ → $\operatorname{sen} x \in \mathbb{R}$ × $\operatorname{sen} x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$= \{ x \in [-1, 1] : \arccos x \in \mathbb{R} \}$

$= [-1, 1]$

2) Lei

$\forall x \in \mathbb{R}$ $\quad \operatorname{sen}^2 x + \cos^2 x = 1$

$\operatorname{sen} x = \pm \sqrt{1 - \cos^2 x}$

$\operatorname{sen} \beta = \pm \sqrt{1 - \cos^2 \beta}$

$\operatorname{sen}(\arccos x) = \pm \sqrt{1 - x^2}$

b) $h(x) = \operatorname{tg}(\arccos x) = \frac{\sqrt{1-x^2}}{x}$

Notemos que: $h(x) = \operatorname{tg}(\arccos x)$

$= \operatorname{tg} x \circ \arccos x$

1) Domínio

$D = \{ x \in D_{\arccos} : \arccos \in D_{\operatorname{tg}} \}$

$= \{ x \in [-1, 1] : \arccos \in \mathbb{R} : x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \} = [-1, 1] \setminus \{-\frac{\pi}{2}, \frac{\pi}{2}\}$

Lei

$\forall x \in \mathbb{R}$

18-

e) $\arccos x + \arccos(-x) = \pi$

$\cos(\arccos x + \arccos(-x)) = \cos\pi$

$\Leftrightarrow \cos(\arccos x)\cos(\arccos(-x)) - \operatorname{sen}(\arccos x)\operatorname{sen}(\arccos(-x)) = -1$

$\Leftrightarrow x \times (-x) - \sqrt{1-x^2} \times \sqrt{1-(-x)^2} = -1$

$\Leftrightarrow -x^2 + (\sqrt{1-x^2})^2 = -1 \Leftrightarrow -x^2 - 1 + x^2 = -1$

$\Leftrightarrow -1 = -1$

d)

19-

a) $2^{\sqrt{3}} < 4^{\sqrt{2}}$

$4^{\sqrt{2}} = 2^{\sqrt{8}}$

b) $8^{\sqrt{\pi}} < 2^{3\sqrt{\pi}}$

$2^{3\sqrt{\pi}}$

c) $2^{\sqrt{2}} \quad 3^{\sqrt{2}}$

d) $3^{\sqrt{2}} < 9^{\frac{1}{\sqrt{3}}}$

$3^{\frac{3}{\sqrt{3}}} = 3^{\sqrt{3}}$

20-

$\log_b c = b$

$\log_c b = \frac{1}{b}$

24-

a) $3^y = 2^{y+1}$

$\Leftrightarrow \log_3(3^y) = \log_3(2^{y+1}) \Leftrightarrow y = \log_3(2^{y+1}) \Rightarrow$

$\Rightarrow y = (y+1)\log_3 2 \Rightarrow y = y\log_3 2 + \log_3 2$

$\Rightarrow y - y\log_3 2 = \log_3 2 \Rightarrow y(1-\log_3 2) = \log_3 2$

$\Rightarrow y = \frac{\log_3 2}{1-\log_3 2}$

b) $e^{2x} + 2e^x - 3 = 0 \Rightarrow (e^x)^2 + 2e^x - 3 = 0$ Seja $e^x = y$

$\Rightarrow y^2 + 2y - 3 = 0 \Rightarrow y = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-3)}}{2 \times 1} \Rightarrow$

$\Rightarrow y = \frac{-2 \pm \sqrt{16}}{2} \Rightarrow y = -3 \vee y = 1$

$\Rightarrow \underbrace{e^x = -3}_{\text{Imp.}} \vee e^x = 1 \Leftrightarrow x = \text{Imp} \vee x = 0$

c) $e^{3x} - 2e^{-x} = 0 \Rightarrow e^{-x}(e^{4x} - 2) = 0$

$\Rightarrow e^{-x} \neq 0 \vee e^{4x} - 2 = 0 \Rightarrow x = \emptyset \vee e^{4x} = 2$

$\Rightarrow x = \emptyset \vee 4x = \ln 2 \Rightarrow x = \emptyset \vee x = \frac{\ln 2}{4}$

d) $\log_x 5 = 0 \Rightarrow$ Imp

e) $\log_2(x^2) = 4 \Rightarrow \log_2(x^2) = \log_2 2^4$

$\Rightarrow x^2 = 16 \Rightarrow x = +4 \vee x = -4$

f) $\ln(10 \ln y) = \ln(5x)$

$\Rightarrow 10 \ln y = 5x$

$\Rightarrow \ln y = \frac{5x}{10} \Leftrightarrow \ln y = \frac{1}{2}x \Rightarrow y = e^{\frac{1}{2}x}$

25-

a) $\cosh^2 x - \operatorname{senh}^2 x = 1$

$\Rightarrow \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$

$\Rightarrow \frac{(e^x)^2 + 2e^x e^{-x} + (e^{-x})^2 - \left((e^x)^2 - 2e^x e^{-x} + (e^{-x})^2\right)}{4}$

$\Rightarrow \frac{e^{2x} - e^{2x} + 2 + 2 + e^{-2x} - e^{-2x}}{4} = \frac{4}{4} = 1$ c.q.m

b) $\cosh x + \operatorname{senh} x = e^x$

$\Rightarrow \frac{e^x + e^{-x} + e^x - e^{-x}}{2} = \frac{2e^x}{2} = e^x$

c) $\operatorname{senh}(-x) = \frac{e^{-x} - e^x}{2}$

$-\operatorname{senh}(x) = -\left(\frac{e^x - e^{-x}}{2}\right) = \frac{e^{-x} - e^x}{2}$)=

d) $\cosh(-x) = \cosh x$

26-

a) $2\cosh(\ln x) = 2\left(\frac{e^{\ln x} + e^{-\ln x}}{2}\right) = e^{\ln x} + e^{-\ln x} =$

$= x + e^{\ln x^{-1}} = x + x^{-1} = x + \frac{1}{x} = \frac{x^2+1}{x}$

b) $\operatorname{senh}(2\ln x) = \frac{e^{2\ln x} - e^{-2\ln x}}{2} = \frac{x^2 - x^{-2}}{2} =$

$= \frac{x^2 - \frac{1}{x^2}}{2} = \frac{\frac{x^4-1}{x^2}}{2} = \frac{x^4-1}{2x^2}$

c) $2\cosh(5x) + \operatorname{senh}(5x) = 2\left(\frac{e^{5x} + e^{-5x}}{2}\right) + \frac{e^{5x} - e^{-5x}}{2}$

$= \frac{2e^{5x} + 2e^{-5x} + e^{5x} - e^{-5x}}{2} = \frac{3e^{5x} + e^{-5x}}{2}$

d) $\ln(\cosh x + \operatorname{senh} x) + \ln(\cosh x - \operatorname{senh} x)$

$= \ln((\cosh x + \operatorname{senh} x)(\cosh x - \operatorname{senh} x))$

$= \ln(\cosh^2 x - \operatorname{senh}^2 x) = \ln 1 = 0$

27-

a)