→ Calculo - ficha 5

2-

a) • Os pontos críticos de F sendo F derivável em I são aqueles que verificam a condição:

$$F'(x) = 0 \Leftrightarrow f(x) = 0$$

$$\Rightarrow x = 1 \vee x = 3$$

b)

		0		1		3		$+\infty$
$f = F'$		−		0	+	0	−	
F		↘		mín	↗	máx	↘	

3-

a) $\int f(x)\,dx$ $\qquad f(x) = \frac{d}{dx}(1-\sqrt{x})$

$= -\frac{1}{2} \times x^{-\frac{1}{2}}$ $\qquad -x^{\frac{1}{2}}$

$\Rightarrow \int -\frac{1}{2} \times x^{-\frac{1}{2}}\,dx = -\sqrt{x} + c$

b) $\int g(x)\,dx$ $\qquad g(x) = \frac{d}{dx}(x+2)$

$= 1$

$\Rightarrow \int 1\,dx = x + c$

c) $\int [f(x) - g(x)]\,dx$

$\Rightarrow \int \left[-\frac{1}{2}x^{-\frac{1}{2}}\right] - \int [1] = -\sqrt{x} + x + c$

4-

a) $\int \frac{1}{(x+1)^2}\,dx = \int (x-1)^{-2}\,dx$

b)

5-

a) $\int x \operatorname{sen} x\,dx = \frac{x^2}{2}\operatorname{sen} x + C$ = Falso

6-

a) $f(x) = 6x \Rightarrow \int f(x) = 3x^2$

$f(x) = x^7 \Rightarrow \int f(x) = \frac{x^8}{8}$

$f(x) = x^5 - 3x + 8 \Rightarrow \int f(x) = \frac{x^6}{6} - \frac{3}{2}x^2 + 8x$

7-

a) $\int (3x^2 - 2x^5)\,dx = \int 3x^2 - \int 2x^5 = x^3 - \frac{1}{3}x^6 + C \quad C \in \mathbb{R}$

b) $\int (\sqrt{x} + 2)^2\,dx = \int x + 4\sqrt{x} + 4\,dx = \int x + \int 4\sqrt{x} + \int 4\,dx$

$= \frac{x^2}{2} + 4\left(\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right) + 4x = \frac{x^2}{2} + 4\,\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 4x = \frac{x^2}{2} + \frac{8}{3}x^{\frac{3}{2}} + 4x$

c) $\int (2x+10)^{20} = \frac{1}{2} \times \int 2(2x+10)^{20} = \frac{1}{2} \times \frac{(2x+10)^{20+1}}{20+1} =$

$= \frac{1}{2} \times \frac{(2x+10)^{21}}{21} = \frac{(2x+10)^{21}}{42}$

d) $\int x^4 (x^5+10)^9\,dx =$

d) $\int x^4 (x^5+10)^9 \, dx \quad \rightarrow \int u' u^\alpha \, dx = \frac{u^{\alpha+1}}{\alpha+1} + c \ (\alpha \neq -1)$

$= \frac{1}{5} \int 5x^4 (x^5+10)^9 = \frac{1}{5} \frac{(x^5+10)^{9+1}}{9+1} = \frac{1}{5} \times \frac{(x^5+10)^{10}}{10} + c, \ c \in \mathbb{R}$

$e^x = e^x$

e) $\int y^2 e^{y^3} = \quad \rightarrow (e^u)' = u' e^u$

$= \frac{1}{3} \int 3y^2 e^{y^3} = \frac{1}{3} e^{y^3} + c, \ c \in \mathbb{R}$

f) $\int \sqrt{2x+1} = \int (2x+1)^{\frac{1}{2}} \quad \rightarrow \int u' u^\alpha \, dx = \frac{u^{\alpha+1}}{\alpha+1} + c \ (\alpha \neq -1)$

$= \frac{1}{2} \times \int 2(2x+1)^{\frac{1}{2}} = \frac{1}{2} \times \frac{(2x+1)^{\frac{3}{2}}}{\frac{3}{2}} = \frac{(2x+1)^{\frac{3}{2}}}{3} + c, \ c \in \mathbb{R}$

g) $\int (1 + tg^2 \alpha) \, dx \quad \rightarrow tg^2 \alpha = sec^2 \alpha - 1$

$\Rightarrow \int 1 + \int tg^2 x = x + \int sec^2 x - 1 = x + tg \, x - x$

$= tg \, x + c, \ c \in \mathbb{R}$

h) $\int \frac{t}{3-t^2} \, dt = -\frac{1}{2} \int \frac{-2t}{3-t^2} \, dt \quad \rightarrow \int \frac{u'}{u} \, dx = \ln|u| + c$

$= -\frac{1}{2} \ln|3-t^2| + c \quad , c \in \mathbb{R}$

i) $\int \frac{1}{4-3x} \, dx = -\frac{1}{3} \int \frac{-3}{4-3x} \, dx \quad \rightarrow \int \frac{u'}{u} \, dx = \ln|u| + c$

$= -\frac{1}{3} \ln|4-3x| + c \quad , c \in \mathbb{R}$

8-

$F : x^5 F'(x) + x^3 + 2x = 3$

$\Rightarrow F'(x) = \frac{3}{x^5} - \frac{x^3}{x^5} - \frac{2x}{x^5}$

$\Rightarrow F'(x) = 3x^{-5} + x^{-2} - 2x^{-4}$

$\Rightarrow F(x) = \int 3x^{-5} - 2x^{-4} - x^{-2}$

$= 3 \times \frac{x^{-4}}{-4} - 2\left(\frac{x^{-3}}{-3}\right) - \left(\frac{x^{-1}}{-1}\right)$

$= -\frac{3}{4}x^{-4} + \frac{2}{3}x^{-3} + \frac{1}{x} + C, \quad C \in \mathbb{R}$

$\int f'(x)\, g(x)\, dx = f(x)\, g(x) - \int f(x) \times g'(x)$

10-

a) $\int x \operatorname{sen} x \, dx$

$= -\cos x \cdot x - \int -\cos x \times 1$

$= -x\cos x - \int -\cos x$

$= -x\cos x + \int \cos x = -x\cos x + \operatorname{sen} x + C, \quad C \in \mathbb{R}$

c.aux

$f'(x) = \operatorname{sen} x \Rightarrow \int f'(x) = -\cos x$

$g(x) = x \Rightarrow g'(x) = 1$

b) $\int x \cos x \, dx =$

$= \operatorname{sen} x \times x - \int \operatorname{sen} x \times 1$

$= x \operatorname{sen} x - \int \operatorname{sen} x$

$= x \operatorname{sen} x + (-\cos x) = x \operatorname{sen} x + \cos x + C, \quad C \in \mathbb{R}$

c.aux

$f'(x) = \cos x \Rightarrow \int f'(x) = \operatorname{sen} x$

$g(x) = x \Rightarrow g'(x) = 1$

c) $\int \ln(1-x)\,dx = \int 1 \times \ln(1-x)$

$= x \times \ln(1-x) - \int x \times \left(\frac{-1}{1-x}\right)$

$= x\ln(1-x) - \int \frac{-x}{1-x}$

Cioux

$f'(x) = 1 \Rightarrow \int f'(x) = x$

$g(x) = \ln(1-x) \Rightarrow g'(x) = \frac{-1}{(1-x)}$

$\rightarrow \int f'(x)\,g(x) = f(x)\,g(x) - \int f(x) \cdot g'(x)$

d) $\int x^2 \operatorname{sen} x\,dx =$

$= -\cos x \cdot x^2 - \int (-\cos x)\,2x$

$= -x^2\cos x + \int 2x\cos x$

$= -x^2\cos x + 2\int x\cos x$

$= -x^2\cos x + 2\left(x\operatorname{sen}x - \int \operatorname{sen}x\right)$

$= -x^2\cos x + 2\left(x\operatorname{sen}x - (-\cos x)\right)$

$= -x^2\cos x + 2x\operatorname{sen}x + 2\cos x + C, \quad C \in \mathbb{R}$

e. aux

$f'(x) = \operatorname{sen}x \Rightarrow \int f'(x) = -\cos x$

$g(x) = x^2 \Rightarrow g'(x) = 2x$

②

$f'(x) = \cos x \Rightarrow \int f'(x) = \operatorname{sen}x$

$g(x) = x \Rightarrow g'(x) = 1$

e) $\int x \operatorname{sen}x \cos x\,dx$

$= \frac{1}{2}\left(\int x\, \underbrace{2\operatorname{sen}x\cos x}_{\operatorname{sen}(2x)}\,dx\right)$

$= \frac{1}{2}\left(\int x \operatorname{sen}(2x)\,dx\right)$

$= \frac{1}{2}\left(\left(-\frac{1}{2}\cos(2x)\cdot x\right) - \int -\frac{1}{2}\cos(2x)\cdot 1\right)$

$= -\frac{1}{4}x\cos(2x) - \frac{1}{4} \cdot \frac{1}{2}\int x(\cos kx)$

e. aux

$f'(x) = \operatorname{sen}(2x)$

$\int f'(x) = \int \operatorname{sen}(2x)$

$= \frac{1}{2} \cdot \int 2\operatorname{sen}(2x)$

$= -\frac{1}{2}\cos(2x)$

$g(x) = x \Rightarrow g'(x) = 1$

f) $\int \ln^2 x = \int 1 \cdot \ln^2 x$ $\rightarrow \int f'(x)g(x) = f(x)g(x) - \int f(x)g'(x)$

$= x \times \ln^2 x - \int x \frac{2\ln x}{x}$

$= x\ln^2 x + \int 2\ln x$

$= x\ln^2 x - \left((2x \times \ln x) - \int 2x \times \frac{1}{x} \right)$

$= x\ln^2 x - 2x\ln x + \int 2$

$= x\ln^2 x - 2x\ln x + 2x$

$= x(\ln^2 x - 2\ln x + 2) + C, \; C \in \mathbb{R}$

C. aux

$f'(x) = 1 \Rightarrow f(x) = x$

$g(x) = \ln^2 x \Rightarrow g'(x) = 2 \times \frac{\ln x}{x}$

②

$f'(x) = 2 \Rightarrow f(x) = 2x$

$g(x) = \ln x \Rightarrow g'(x) = \frac{1}{x}$

g) $\int e^x \cos x \, dx$

$= \sin x \times e^x - \int \sin x \, e^x$

$= e^x \sin x - \int e^x \sin x$

$= e^x \sin x - \left(-\cos x \times e^x - \int e^x (-\cos x) \right)$

$= e^x \sin x + e^x \cos x + \int e^x \cos x$

• Soit $I = \int e^x \cos x$

$I = e^x \sin x + e^x \cos x - I \Rightarrow$

$\Rightarrow 2I = e^x \sin x + e^x \cos x \Rightarrow I = \frac{1}{2}(e^x \sin x + e^x \cos x)$

$\Rightarrow \int e^x \cos x = \frac{e^x}{2}(\sin x + \cos x)$

C. aux

$f'(x) = \cos x \Rightarrow f(x) = \sin x$

$g(x) = e^x \Rightarrow g(x) = e^x$

②

$f'(x) = \sin x \Rightarrow f(x) = -\cos x$

$g(x) = e^x \Rightarrow g'(x) = e^x$

11 –

a) $\int x\sqrt{x-1}\,dx \qquad x = t^2+1$ $\rightarrow \int f(x)dx = \int f(e(t)) \cdot e'(t)\,dt$

$= \int t^2+1\left(\sqrt{t^2+1}-1\right) \times (t^2+1)' = \int (t^4+1)t \times 2t = \int 2t^4 + 2t^3$

$= \frac{2}{5} + \int \frac{5}{2} \times 2t^4 + \frac{2}{4}\int 4t^3 = \frac{2}{5}t^5 + \frac{1}{2}t^4$

b) $\int \sqrt{1-x^2}\,dx \qquad x = \operatorname{sen} t$

$= \int \sqrt{1-\operatorname{sen}^2 t} \times \cos t = \int \cos^2 t \cdot \cos t = \int \cos^2 t$

$= \frac{1}{2}\int 1+\cos(2t)\,dt = \frac{1}{2}\left[t + \frac{1}{2}\operatorname{sen}(2t)\right]dt$

$= \frac{1}{2}\left[t + \operatorname{sen} t \cos t\right] + c = \underset{t = \operatorname{arcsen} x}{=} \frac{1}{2}\left[\operatorname{arcsen} x + \operatorname{sen}(\operatorname{arcsen} x) \cdot \cos(\operatorname{arcsen} x)\right] + c$

$= \frac{1}{2}\left[\operatorname{arcsen} x + x \cdot \sqrt{1-x^2}\right] + c, \quad c \in \mathbb{R}$

e) $\int \frac{e^{2x}}{1+e^x}\,dx \qquad x = \ln t$

$= \int \frac{e^{2\ln t}}{1+e^{\ln t}} \times \frac{1}{t} \Rightarrow \int \frac{t^2}{1+t} \cdot \frac{1}{t} = \int \frac{t^2}{t+t^2} = \int \frac{t}{1+t}$

11 -

d) $\int \sqrt{1+x^2}\,dx \quad x = \operatorname{senh} t$

$$\int f(x)\,dx = \int [f(\varphi(t))]\,\varphi'(t)\,dt \qquad x = \varphi(t)$$

$= \int \sqrt{1+\operatorname{senh}^2 t} \times \cosh t$

$x' = (\operatorname{senh} t)' = \cosh t$

$1+\operatorname{senh}^2 t = \cosh^2 t$

$= \int \sqrt{\cosh^2 t} \times \cosh t$

$= \int \cosh t \times \cosh t = \int \cosh^2 t$

$\Rightarrow \int f'(x)\,g(x)\,dx = f(x)g(x) - \int f(x) \times g'(x)$

e.cuux

$= \int \cosh t \times \cosh t =$

$f'(x) = \cosh t \Rightarrow f(x) = \operatorname{senh} t$

$g(x) = \cosh t \Rightarrow g'(x) = \operatorname{senh} t$

$= \operatorname{senh} t \times \cosh t - \int \operatorname{senh}^2 t$

12-

a) $\int \frac{3x^2-4x+1}{(x^2-1)(x-2)}\,dx = \int \frac{-\frac{1}{3}}{x-1} + \int \frac{-3}{(x+1)} + \int \frac{\frac{19}{3}}{x-2}$

$= -\frac{1}{3}\int \frac{1}{x-1} - 3\int \frac{1}{x+1} + \frac{19}{3}\int \frac{1}{x-2} = -\frac{1}{3}\ln|x-1| - 3\ln|x+1| + \frac{19}{3}\ln|x-2|$

Le aux

$\frac{3x^2-4x+1}{(x^2-1)(x-2)} = \frac{3x^2-4x-1}{(x-1)(x+1)(x-2)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x-2}$

$= \frac{A(x+1)(x+2)}{(x-1)(x+1)(x+2)} + \frac{B(x-1)(x+2)}{(x-1)(x+1)(x-2)} + \frac{C(x+1)(x-1)}{(x+1)(x-1)(x+2)}$

$\Rightarrow \frac{3x^2-4x-1}{(x-1)(x+1)(x+2)} = \frac{A(x^2+2x+x+2)}{(x-1)(x+1)(x+2)} + \frac{B(x^2+2x-x-2)}{(x-1)(x+1)(x-2)} + \frac{Cx^2-C}{(x-1)(x+1)(x+2)}$

$\Rightarrow \frac{3x^2-4x-1}{(x-1)(x+1)(x+2)} = \frac{Ax^2+3Ax+2A+Bx^2+Bx-2B+Cx^2-C}{(x-1)(x+1)(x+2)}$

$\Rightarrow 3x^2-4x-1 = Ax^2+Bx^2+Cx^2+3Ax+Bx+2A-2B-C$

$\Rightarrow 3x^2-4x-1 = x^2(A+B+C)+x(3A+B)+2A-2B-C$

$\Rightarrow \begin{cases} A+B+C=3 \\ 3A+B=-4 \\ 2A-2B-C=-1 \end{cases} \Rightarrow \begin{cases} A-4-3A+C=3 \\ B=-4-3A \\ 2A-2(-4-3A)+C=-1 \end{cases} \Rightarrow \begin{cases} -2A+C=7 \\ 2A+8+6A-C=-1 \end{cases}$

$$\begin{cases} C = 7 + 2A \\ B = -4 - 3A \\ 8A - (7 + 2A) = -9 \end{cases} \Rightarrow \begin{cases} \\ \\ 6A = -2 \end{cases} \Rightarrow \begin{cases} C = 7 + \left(-\frac{2}{3}\right) \\ B = -4 - 3\left(-\frac{1}{3}\right) \\ A = -\frac{1}{3} \end{cases}$$

$$\Rightarrow \begin{cases} C = \frac{19}{3} \\ B = -3 \\ A = -\frac{1}{3} \end{cases}$$

b) $$\int \frac{2x^2 + x + 1}{(x-1)(x+1)^2} = \int \frac{2x^2 + x + 1}{(x-1)(x+1)(x+1)}$$

Produs

$$\frac{2x^2 + x + 1}{(x-1)(x+1)(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x+1)} + \frac{C}{x+1}$$

$$= \frac{A(x+1)(x+1)}{(x-1)(x+1)(x+1)} + \frac{B(x-1)(x+1)}{(x-1)(x+1)(x+1)} + \frac{C(x+1)(x-1)}{(x-1)(x+1)(x+1)}$$

$$\frac{2x^2 + x + 1}{(x-1)(x+1)(x+1)} = \frac{Ax^2 + 2Ax + A + Bx^2 - B + Cx^2 - C}{(x-1)(x+1)(x+1)}$$

$$\Rightarrow 2x^2 + x + 1 = Ax^2 + Bx^2 + Cx^2 + 2Ax + A - B - C$$

$$\Rightarrow 2x^2 + x + 1 = x^2(A + B + C) + 2A(x) + A - B - C$$

$$\begin{cases} A + B + C = 2 \\ 2A = 1 \\ A - B - C = 1 \end{cases} \Rightarrow \begin{cases} B + C = \frac{3}{2} \\ A = \frac{1}{2} \\ -B - C = \frac{1}{2} \end{cases} \Rightarrow \begin{cases} \\ \\ B = \frac{1}{2} - C \end{cases}$$

12- e) $\int \frac{4x^2+x+1}{x^3-x} = \int \frac{4x^2+x+1}{x(x^2-1)} = \int \frac{4x^2+x+1}{x(x-1)(x+1)}$

$\Rightarrow \int \frac{1}{x} + \int \frac{2}{x-1} + \int \frac{1}{(x+1)} = \ln|x| + 2\ln|x-1| + \ln|x+1|$

e.ewX

$\frac{4x^2+x+1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$

$\Rightarrow \frac{4x^2+x+1}{x(x-1)(x+1)} = \frac{A(x-1)(x+1)}{x(x-1)(x+1)} + \frac{B(x)(x+1)}{(x-1)x(x+1)} + \frac{C(x)(x-1)}{x(x+1)(x-1)}$

$\Rightarrow 4x^2+x+1 = Ax^2 - A + Bx^2 + Bx + Cx^2 - Cx$

$\Rightarrow 4x^2+x-1 = x^2(A+B+C) + x(B-C) + A$

$\Rightarrow \begin{cases} A+B+C=4 \\ B-C=1 \\ -A=-1 \end{cases} \Rightarrow \begin{cases} B+1=3-C \\ 3-C-C=1 \\ A=1 \end{cases} \Rightarrow \begin{cases} \\ -2C=-2 \\ \end{cases}$

$\Rightarrow \begin{cases} B=2 \\ C=1 \\ A=1 \end{cases}$

d) $\int \frac{27}{x^4-3x^3}\,dx = \int \frac{27}{x^3(x-3)} = \int \frac{x^2+3x+1}{x^3} - \int \frac{1}{x-3}$

$= \int (x^2+3x-1)\,x^{-3} - \ln|x-3|$

$= \int x^{-1} + 3x^{-2} - x^{-3} - \ln|x-3| = \ln|x| +$

Coeficientes

$\frac{27}{x^3(x-3)} = \frac{Ax^2+Bx+C}{x^3} + \frac{D}{x-3} = \frac{Ax^2+Bx+C(x-3)+Dx^3}{x^3(x-3)}$

$= Ax^3 + Bx^2 + Cx - 3Ax^2 - 3Bx - 3C + Dx^3$

$= x^3(A+D) + x^2(B-3A) + x(C-3B) - 3C$

$\left.\begin{array}{l} A+D=0 \\ B-3A=0 \\ C-3B=0 \\ -3C=27 \end{array}\right\} \Rightarrow \left.\begin{array}{l} D=-1 \\ A=1 \\ B=3 \\ C=-9 \end{array}\right.$

e) $\int \frac{x^4-8}{x^3-2x^2} = \int \frac{x^4-8}{x^2(x-2)}$

Coeficientes

$\frac{x^4-8}{x^2(x-2)} = \frac{Ax+B}{x^2} + \frac{C}{x-2}$

$\frac{x^4-8}{x^2(x-2)} = \frac{Ax^2-2Ax+Bx-2B+Cx^2}{x^2(x-2)}$

$\Rightarrow$

13 -

a) $\int \frac{2x}{x^2-1} dx = \frac{1}{2} \int \frac{2x}{x^2-1} = \ln|x^2-1| + C$, $C \in \mathbb{R}$

b) $\int \frac{x}{\sqrt{2x^2-1}} dx = \frac{1}{2} \int \frac{2x}{\sqrt{x^2-1}} = \operatorname{argcosh}(x^2-1) + C$, $C \in \mathbb{R}$

c) $\int \frac{1}{x} \operatorname{sen}(\ln x)\, dx = -\cos(\ln x) + C$, $C \in \mathbb{R}$

d) $\int \frac{-3}{x(\ln x)^3} = -3(\ln x)^{-3x} = \frac{x^{-3x}}{\ln x} + C$, $C \in \mathbb{R}$

e) $\int \frac{e^x}{1+e^{2x}} = \operatorname{arctg}(e^x) + C$, $C \in \mathbb{R}$

f) $\int \frac{e^x}{1-2e^x} = -\frac{1}{2}\int\left(\frac{-2e^x}{1-2e^x}\right) = -\frac{1}{2}\ln|1-2e^x| + C$, $C \in \mathbb{R}$

g) $\int \frac{1}{\cos^2(7x)} dx = \int \sec^2(7x)\, dx = \frac{1}{7}\int 7\sec^2(7x)\, dx = \operatorname{tg}(7x) + C$, $C \in \mathbb{R}$

h) $\int \left(\sqrt{2x-1} - \sqrt{1-3x}\right) = \int \sqrt{2x-1} - \int \sqrt{1-3x}$

$= \frac{1}{2}\int 2(2x-1)^{\frac{1}{2}} - \left(-\frac{1}{3}\right)\int -3(1-3x)^{\frac{1}{2}}$

$= \frac{1}{2} \cdot \frac{(2x-1)^{\frac{3}{2}}}{\frac{3}{2}} - \left(-\frac{1}{3}\,\frac{(1-3x)^{\frac{3}{2}}}{\frac{3}{2}}\right) =$

j) $\int \tgh x \, dx = \ln(\cosh x) + C$ $\rightarrow \int u' \tgh u \, dx = \ln(\cosh u) + C$

$e^x = x$

k) $\int \frac{1}{e^{3x}} dx = \int e^{3x} = -\frac{1}{3} \int -3 e^{-3x} = -\frac{1}{3e^{3x}}$

l) $\int \frac{-7}{\sqrt{1-5x}} dx = -7 \int (1-5x)^{-\frac{1}{2}} = -7\left(-\frac{1}{5}\right) \int -5 (1-5x)^{-\frac{1}{2}}$

$= \frac{7}{5} \times \frac{(1-5x)^{\frac{1}{2}}}{\frac{1}{2}} = \frac{14}{5}(1-5x)^{\frac{1}{2}} + C, \; C \in \mathbb{R}$

m) $\int \frac{\sqrt{1+3\ln x}}{x}$

n) $\int x \sen^2 x^2 = \frac{1}{2} \int 2x \sen x^2 = -\cos x^2 + C \quad C \in \mathbb{R}$

$\rightarrow \int u' \sen u \, dx = -\cos u + C$

o) $\int \frac{1}{x(\ln^2 x + 1)} dx =$

p) $\int \left(\frac{2}{x} - 3\right)^2 \frac{1}{x^2}\, dx = \int \left(\frac{4}{x^2} - \frac{12}{x} + 9\right) \frac{1}{x^2}\, dx = \int \frac{4}{x^4} - \frac{1}{x^3} + \frac{9}{x^2}\, dx$

$= \int 4x^{-4} - \int x^{-3} + \int 9x^{-2} = \frac{4}{3} \cdot \int \frac{3}{4} 4x^{-4} - \left(-\frac{1}{2}\right) \int (-2) x^{-3} + 9 \cdot \left(-\frac{1}{9}\right) \int 9x^{-2}$

$= -\frac{4}{3} x^{-3} + \frac{1}{2} x^{-2} - 9x^{-1} + C \quad , C \in \mathbb{R}$

q) $\int \operatorname{sen}(\pi - 2x)\, dx$ $\rightarrow \int \operatorname{sen} u\, dx = -\cos u + C$

$= -\frac{1}{2} \int -2 \operatorname{sen}(\pi - 2x)\, dx = -\frac{1}{2} \left(-\cos(\pi - 2x)\right) = \frac{1}{2} \cos(\pi - 2x)$

14-

a) $\int_0^1 \sqrt{2-x^2}$

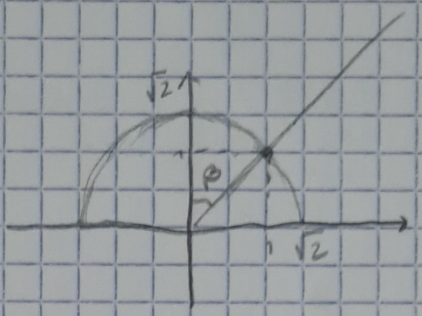
Temos

$f(x) = \sqrt{2-x^2}$

$\Leftrightarrow y = \sqrt{2-x^2} \quad > 0$

$\Leftrightarrow y^2 = 2 - x^2$

$\Leftrightarrow x^2 + y^2 = 2$



- $\int_0^1 \sqrt{2-x^2}$ - area limitada pelo grafico de $y = \sqrt{2-x^2}$ quando

$0 \leq x \leq 1$ = area ◔ + area △

$$= \frac{\pi r^2}{2} + \frac{b \times b}{2}$$

$$= \frac{\frac{\pi}{4}(\sqrt{2})^2}{2} + \frac{1 \times 1}{2}$$

$$= \frac{\pi}{4} + \frac{1}{2} \quad \text{c.q.m.}$$

b) $\int_0^2 \sqrt{4-x^2}$

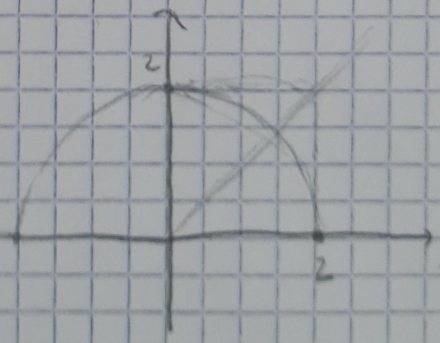
Temos

$f(x) = \sqrt{4-x^2}$

$\Leftrightarrow y = \sqrt{4-x^2}$

$\Leftrightarrow y^2 = 4 - x^2$

$\Leftrightarrow y^2 + x^2 = 4$



- $\int_0^2 \sqrt{4-x^2}$ - area limitada pelo grafico $y = \sqrt{4-x^2}$ quando

$0 \leq x \leq 2$ = area □ +

21-

a) $\int_{-1}^{2} x^3\,dx = \left.\frac{x^4}{4}\right|_{-1}^{2} = \frac{2^4}{4} - \frac{(-1)^4}{4} = 4 - \frac{1}{4} = \frac{15}{4}$

b) $\int_0^{\pi} x\cos x = x\operatorname{sen}x + \cos x \Big|_0^{\pi} = (\pi \operatorname{sen}\pi + \cos\pi) - (0\times\operatorname{sen}0 + \cos 0)$

$= -1 - (1) = -2$

P. aux

$\rightarrow \int f'(x)\,g(x) = f(x)g(x) - \int f(x)\,g'(x)$

$\int x\cos x = x\operatorname{sen}x - \int \operatorname{sen}x \times 1$

$= x\operatorname{sen}x - (-\cos x)$

$= x\operatorname{sen}x + \cos x$

P. aux

$f'(x) = \cos x \Rightarrow f(x) = \operatorname{sen}x$

$g(x) = x \Rightarrow g'(x) = 1$

c) $\int_{\pi}^{2\pi} \frac{\operatorname{sen}x}{x}\,dx = ?$

$\left(\frac{1}{x}\right)' = \frac{1\times x - x\cdot 1}{x^2} = \frac{0-1}{x^2}$

P. aux

$\rightarrow \int f'(x)\,g(x) = f(x)g(x) - \int f(x)\,g'(x)$

$\int \operatorname{sen}x \times \frac{1}{x} = (-\cos x)\times\frac{1}{x} - \int (-\cos x)\times\frac{1}{x^2}$

$= -\frac{\cos x}{x} - \int -\frac{\cos x}{x^2}$

P. aux

$f'(x) = \operatorname{sen}x \Rightarrow f(x) = -\cos x$

$g(x) = \frac{1}{x} \Rightarrow g'(x) = \frac{1}{x^2}$

$\int \frac{1}{x}\times\operatorname{sen}x = \ln|x|\cdot\cos x - \int \cos x \ln|x|$

$\rightarrow$ impr.

P. aux

$f'(x) = \frac{1}{x} \Rightarrow f(x) = \ln|x|$

$g(x) = \operatorname{sen}x \Rightarrow g'(x) = \cos x$

25-

a) $\int_0^1 x\,dx = \frac{x^2}{2}\Big|_0^1 = \frac{1}{2} - 0 = \frac{1}{2}$

b) $\int_2^3 e^x\,dx = e^x\Big|_2^3 = e^3 - e^2$

c) $\int_0^\pi \operatorname{sen} x\,dx = (-\cos x)\Big|_0^\pi = -\cos\pi - (-\cos 0) = 1 - (-1) = 2$

d) $\int_0^1 \frac{1}{1+x^2} = \operatorname{arctg} x\Big|_0^1 = \operatorname{arctg} 1 - \operatorname{arctg} 0 = \frac{\pi}{4} - 0 = \frac{\pi}{4}$

e) $\int_0^{\frac{1}{5}} \frac{dt}{\sqrt{3-5t}} = \int_0^{\frac{1}{5}} (3-5t)^{-\frac{1}{2}} = -\frac{1}{5}\int_0^{\frac{1}{5}} -5(3-5t)^{-\frac{1}{2}} = -\frac{1}{5}\left(\frac{\sqrt{3-5t}}{\frac{1}{2}}\right)\Big|$

$= -\frac{1}{5}\left(2\sqrt{2} - 2\sqrt{3}\right) = \frac{-2\sqrt{2} + 2\sqrt{3}}{5}$

f) $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \operatorname{sen} 3t\,dt = \frac{1}{3}\left(-\cos 3t\right)\Big|_{-\frac{\pi}{3}}^{\frac{\pi}{3}} = \frac{1}{3}\left(-\cos\pi - (-\cos(-\pi))\right)$

$= \frac{1}{3}\left(-(-1) + (-1)\right) = 0$

29-

a) $\int_0^1 \frac{e^x}{\sqrt{e^x+1}} = \int_0^1 e^x (e^x+1)^{-\frac{1}{2}}$ $\rightarrow \int u' u^\alpha dx = \frac{u^{\alpha+1}}{\alpha+1} + C$

$= \frac{(e^x+1)^{\frac{1}{2}}}{\frac{1}{2}} \Big|_0^1 = \frac{\sqrt{e^1+1}}{\frac{1}{2}} - \frac{\sqrt{2}}{\frac{1}{2}} = 2(\sqrt{e^1+1} - \sqrt{2})$

e) $\int_0^2 x^3 e^{x^2} = \int_0^2 x^2 x e^{x^2} = \frac{1}{2} e^x \cdot x^2 \Big|_0^2 - \frac{1}{2} e^x \Big|_0^1$

$= 2e^4 - \left(\frac{1}{2}(e^1 - e^0)\right) = 2e^4 - \frac{1}{2}e^1 + \frac{1}{2}$

$\rightarrow$ C. aux

$\int x^3 x e^x = \frac{1}{2} e^{x^2} \cdot x^2 - \int \frac{1}{2} e^{x^2} \cdot 2x$

$= \frac{1}{2} e^{x^2} \cdot x^2 - \int x e^{x^2}$

$= \frac{1}{2} e^{x^2} \cdot x^2 - \frac{1}{2} e^{x^2}$

$\int f'(x) g(x) = f(x) g(x) - \int f(x) g'(x)$

$f'(x) = x e^{x^2} \Rightarrow f(x) = \frac{1}{2} e^{x^2}$

$g(x) = x^2 \Rightarrow g'(x) = 2x$

29-

b) $\int_0^{\pi} (x+2)\cos x = (x+2)\sin x + \cos x \Big|_0^{\pi} = -1 - (1) = -2$

c. dux

$\int (x+2)\cos x = (x+2)\sin x - \int \sin x$

$= (x+2)\sin x - (-\cos x)$

$= (x+2)\sin x + \cos x$

$\int f'(x)g(x) = f(x)g(x) - \int f(x)g'(x)$

e · dux

$f'(x) = \cos x \Rightarrow f(x) = \sin x$

$g(x) = (x+2) \Rightarrow g'(x) = 1$

d) $\int_0^1 \ln(x^2+1)$

e. dux

$\int \ln(x^2+1)$

$\Rightarrow \int f(x)g(x) = f(x)g(x) - \int f(x)g'(x)$

$f(x) = 1 \Rightarrow f(x) = x$

$g(x) = \ln(x^2+1) \Rightarrow g'(x) = \frac{2x}{x^2+1}$

29- f) $\int_{-\pi/2}^{\pi/2} |\operatorname{sen} x|\,dx$

Ora $|\operatorname{sen} x| = \begin{cases} -\operatorname{sen} x, & \operatorname{sen} x < 0 \\ \operatorname{sen} x, & \operatorname{sen} x \geq 0 \end{cases} = \begin{cases} -\operatorname{sen} x, & -\frac{\pi}{2} \leq x \leq 0 \\ \operatorname{sen} x, & 0 \leq x \leq \frac{\pi}{2} \end{cases}$

Assim,

$$\int_{-\pi/2}^{\pi/2} |\operatorname{sen} x|\,dx = \int_{-\pi/2}^{0} |\operatorname{sen} x|\,dx + \int_{0}^{\pi/2} |\operatorname{sen} x|\,dx$$

$$= \int_{-\pi/2}^{0} -\operatorname{sen} x\,dx + \int_{-\pi/2}^{0} \operatorname{sen} x\,dx$$

$$= \cos x \Big|_{-\frac{\pi}{2}}^{0} + (-\cos x)\Big|_{\frac{\pi}{2}}^{0} = 2$$

h) $\int_{-3}^{2} \sqrt{|x|}\,dx$

Ora $\sqrt{|x|} = \begin{cases} \sqrt{-x}, & x < 0 \\ \sqrt{x}, & x \geq 0 \end{cases} \Rightarrow \begin{cases} \sqrt{-x}, & -3 \leq x \leq 0 \\ \sqrt{x}, & 0 \leq x \leq 2 \end{cases}$

Assim

$$\int_{-3}^{2} \sqrt{|x|}\,dx = \int_{-3}^{0} \sqrt{-x} + \int_{0}^{2} \sqrt{x} = -\frac{(-x)^{\frac{3}{2}}}{\frac{3}{2}}\Big|_{-3}^{0} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\Big|_{0}^{2}$$

$$= 0 + \frac{3^{\frac{3}{2}}}{\frac{3}{2}} + \frac{2^{\frac{3}{2}}}{\frac{3}{2}} = \frac{3^{\frac{3}{2}} + 2^{\frac{3}{2}}}{\frac{3}{2}} = 2\left(\frac{\sqrt{3^3} + \sqrt{2^3}}{3}\right) =$$

$$= 2\frac{(3\sqrt{3})}{3} + 2\frac{(2\sqrt{2})}{2}$$

$$= 2\sqrt{3} + \frac{4\sqrt{2}}{3}$$

Como $\int (+x)^{-\frac{1}{2}}$

31-

a) $\int_{-1}^{1} e^{\arcsen x}\, dx \quad x = \operatorname{sen} t$

derivada mudada!!

$$\int_{-1}^{1} e^{\arcsen(\operatorname{sen} t)} \times \cos t = \int_{-1}^{1} e^t \cos t = \left.\frac{e^t(\cos t + \operatorname{sen} t)}{2}\right|_{-1}^{1}$$

$$= \left.e^{\arcsen x}\left(\frac{\cos(\arcsen x) + x}{2}\right)\right|_{-1}^{1} = 0 - 0 = 0$$

c. aux

$$\int \underbrace{e^t \cos t}_{a} = e^t \cos t - \int e^t(-\operatorname{sen} t)$$

$$= e^t \cos t + \int e^t \operatorname{sen} t$$

$$= e^t \cos t + e^t \operatorname{sen} t - \underbrace{\int e^t \cos t}_{a}$$

$$\Rightarrow a = e^t \cos t + e^t \operatorname{sen} t - a$$

$$\Rightarrow 2a = e^t(\cos t + \operatorname{sen} t)$$

$$a = \frac{e^t(\cos t + \operatorname{sen} t)}{2}$$

$\rightarrow \int f'(x) g(x) = f(x)g(x) - \int f(x) g'(x)$

c. aux

$f'(t) = e^t \Rightarrow f(t) = e^t$

$g(t) = \cos t \Rightarrow g'(t) = -\operatorname{sen} t$

$f'(t) = e^t \Rightarrow f(t) = e^t$

$g(t) = \operatorname{sen} t \Rightarrow g'(t) = \cos t$

d) $\int_{1}^{2} x\sqrt{x-1} \qquad t = x-1$

$$= \int_{0}^{1} (t+1)\sqrt{t}\, dt =$$

$$= \int_{0}^{1} t^{\frac{3}{2}} + t^{\frac{1}{2}}\, dt$$

$$= \int_{0}^{1} \frac{t^{5/2}}{\frac{5}{2}} + \frac{t^{3/2}}{3/2} =$$

$$= \frac{2}{5} + \frac{2}{3} = \frac{16}{15}$$

$\rightarrow \int_b^a g(x)\, dx = \int_\beta^\alpha g(f(x))\, f'(t)\, dt$

onde $\alpha = f(a)$

$\beta = f(b)$

- $t = x-1 \Rightarrow x = t+1 = f(t)$
- $x = 1 \Rightarrow t = 0$
- $x = 2 \Rightarrow t = 1$

33 -

$$\int_1^{+\infty} \frac{1}{x^k}\,dx = \lim_{b\to -\infty} \int_1^b \frac{1}{x^k}$$

- $\boxed{k \in \mathbb{R}}$

  - $k = 0$, $x^k = x^0 = 1$

$$\int_1^{+\infty} 1\,dx = \lim_{b\to+\infty} \int_1^b 1\,dx = \lim_{b\to+\infty} x\Big|_{x=1}^{b} = \lim_{b\to+\infty} [b-1]$$

$= +\infty$ logo não existe e o intervalo impróprio é divergente

- $k = -1$

$$\int_1^{+\infty} \frac{1}{x}\,dx = \lim_{b\to+\infty} \int_1^b \frac{1}{x} = \lim_{b\to\infty} \ln x\Big|_{x=1}^{b} =$$

$$= \lim_{b\to+\infty} \ln b - 0 = +\infty$$ não existe e o intervalo impróprio é divergente

- $k \neq 1$

$$\int_1^{+\infty} \frac{1}{x^{-k}}\,dx = \lim_{b\to+\infty} \int_1^b x^{-k} = \lim_{b\to+\infty} \frac{x^{-k+1}}{-k+1}\Big|_{x=1}^{b}$$

$$= \frac{1}{-k+1}\left(\lim_{b\to+\infty} b^{-k+1} - 1\right) \begin{cases} +\infty & \text{se } -k+1 > 0 \\ 0-1 & \text{se } -k+1 < 0 \end{cases}$$

Em resumo

$$\int_1^{+\infty} \frac{1}{x^k}\,dx \begin{cases} k > 1 \text{ convergente} \\ k \leq 1 \text{ divergente} \end{cases}$$

34- $\int_0^{+\infty} e^{-kx}\,dx = \lim_{b\to+\infty} \int_0^b e^{-kx}\,dx$

- $k \in \mathbb{R}$

$$-\frac{1}{k}\int -k\, e^{-kx} = -\frac{1}{k} e^{-kx} + c, \quad c \in \mathbb{R},\ k \neq 0$$

- $k = 0$

$\int_0^{+\infty} e^0\,dx = \int_0^{+\infty} 1\,dx$ diverge

- $k \neq 0$

$$\int_0^{+\infty} e^{-kx}\,dx = \lim_{b\to+\infty} \int_0^b e^{-kx}\,dx = \lim_{b\to+\infty} \left[-\frac{1}{k} e^{-kx}\right]_{x=0}^{b}$$

$$= -\frac{1}{k} \lim_{b\to+\infty} \left[e^{-kb} - e^0\right]$$

$-k > 0 \Rightarrow k < 0$, o limite não existe e $\int_0^{+\infty} e^{-kx}\,dx$ diverge

$-k < 0 \Rightarrow k > 0$, $\lim_{b\to+\infty} e^{-kb} = 0$ e

$\int_0^{+\infty} e^{-kx}\,dx$ diverge

Em resumo

$\int_0^{+\infty} e^{-kx}\,dx$ $\begin{cases} k \leq 0 \text{ diverge} \\ k > 0 \text{ Converge} \end{cases}$

38- d) $\int_0^1 x \ln x \, dx$

• Este é um integral impróprio do tipo 2 pois

$D_f =]0, +\infty[$, isto é $0 \notin D_f$

Assim $\int_0^1 x \ln x \, dx = \lim_{x \to 0^+} \int_b^1 x \ln x \, dx$

• Ora usando primitivas por partes

$$\int f'(x) g(x) = f(x) g(x) - \int f(x) g'(x)$$

$$\int x \ln x = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \times \frac{1}{x}$$

c. aux

$f'(x) = x \Rightarrow f(x) = \frac{x^2}{2}$

$g(x) = \ln x \Rightarrow g'(x) = \frac{1}{x}$

$$= \frac{x^2}{2} \ln x - \frac{1}{2} \int x$$

$$= \frac{x^2}{2} \ln x - \frac{1}{2} \left(\frac{x^2}{2} \right) = \frac{x^2}{2} \ln x - \frac{x^2}{4}$$

$$= \frac{x^2}{4} (2 \ln x - 1) = \frac{x^2}{4} (\ln x^2 - 1) + C, \; C \in \mathbb{R}$$

Então

$$\int_0^1 x \ln x \, dx = \lim_{b \to 0^+} \left[\frac{x^2}{4} (\ln x^2 - 1) \right]_{x=b}^{1}$$

$$= \lim_{b \to 0^+} \left[\frac{1}{4} (\ln 1^2 - 1) - \frac{b^2}{4} (\ln b^2 - 1) \right]$$

$$= -\frac{1}{4} - \frac{1}{4} \left(\lim_{b \to 0^+} \left[b^2 \ln b^2 - b^2 \right] \right)$$

$$= -\frac{1}{4} - \frac{1}{4} \left(\lim_{b \to 0^+} b^2 \ln b^2 \right) = -\frac{1}{4}$$

c. aux

$$\lim_{x \to 0} x \ln x = \lim_{x \to 0} \frac{\ln x}{\frac{1}{x}} \overset{\text{RH}}{=} \lim_{x \to 0} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x \to 0} (-x) = 0$$

25-

g) $\int_e^{e^2} \frac{(\ln x)^2}{x} = \int_e^{e^2} \frac{1}{x} \times (\ln x)^2$

$\rightarrow \int u' u^a \, dx = \frac{u^{a+1}}{a+1} + C$

$= \frac{(\ln x)^3}{3} \Big|_e^{e^2} = \frac{(\ln e^2)^3}{3} - \frac{(\ln e)^3}{3} = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$

35-

a) $\lim_{b\to+\infty} \int_2^b \frac{1}{x-1} = \lim_{b\to+\infty} \ln|x-1| \Big|_2^b$

$= \lim_{b\to+\infty} \ln|b-1| - \underbrace{\ln|2-1|}_{0}$

$= \lim_{x\to+\infty} \ln|b-1| = +\infty \Rightarrow$ Divergente

b) $\lim_{a\to-\infty} \int_a^0 x e^{-x^2} = \lim_{a\to-\infty} \left(-\frac{1}{2} \int_a^0 -2x\, e^{-x^2}\right) =$

$= \lim_{a\to+\infty} \left(-\frac{1}{2} \left(e^{x^2}\right)\Big|_a^0 = \lim_{a\to-\infty} -\frac{1}{2}\left(1 - e^{-a^2}\right)\right.$

$= \lim_{a\to-\infty} -\frac{1}{2}\left(1 - \underbrace{\frac{1}{e^{a^2}}}_{\frac{1}{0}}\right) = -\frac{1}{2}(1-0) = -\frac{1}{2} \Rightarrow$ convergente

c)

$\lim_{b\to+\infty} \int_2^b \frac{1}{x^2-1} = \lim_{b\to+\infty} \int_2^b \frac{1}{(x-1)(x-1)}$

d) $\lim_{b\to+\infty} \int_1^b x^2 = \lim_{b\to+\infty} \frac{x^3}{3}\Big|_1^b = \lim_{b\to+\infty} \frac{b^3}{3} - \frac{1}{3} = \infty$

$\Downarrow$

Divergente

35-

e) $\lim_{a \to +\infty} \int_1^{+\infty} \frac{1}{x^2} dx = \lim_{a \to +\infty} \int_1^{+\infty} x^{-2}$

$= \lim_{a \to +\infty} \frac{x^{-1}}{-1} \Big|_1^a = \lim_{x \to +\infty} -\frac{1}{a} - \left(-\frac{1}{1}\right) = 1$

f) $\lim_{a \to +\infty} \int_1^a \cos(\pi x) = \lim_{a \to +\infty} \frac{1}{\pi} \int_1^a \pi \cos(\pi x)$

$= \lim_{a \to +\infty} \pi \left(\operatorname{sen}(\pi x)\right)\Big|_1^a = \lim_{a \to +\infty} \pi \left(\operatorname{sen}(\pi a) - \underbrace{\operatorname{sen}\pi}_{0}\right)$

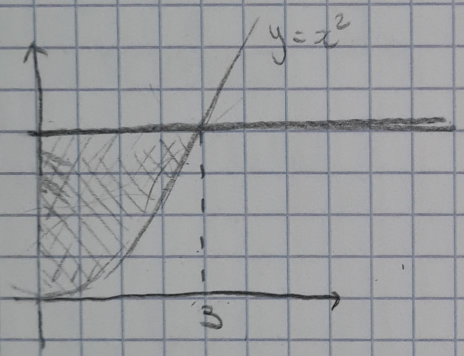
g) $\lim_{a \to +\infty} \int_e^a \frac{1}{x \ln x} = \lim_{a \to +\infty} \int_e^a \frac{1}{x} \cdot (\ln x)^{-1}$

h) $\lim_{b \to -\infty} \int_b^0 \frac{1}{1+x^2} + \lim_{a \to +\infty} \int_0^a \frac{1}{1+x^2}$

$= \lim_{b \to -\infty} \operatorname{arctg} x \Big|_b^0 + \lim_{a \to +\infty} \operatorname{arctg} x \Big|_0^a$

$= 0 + \frac{\pi}{2} + \frac{\pi}{2} - 0 = \pi \Rightarrow$ convergente

45-

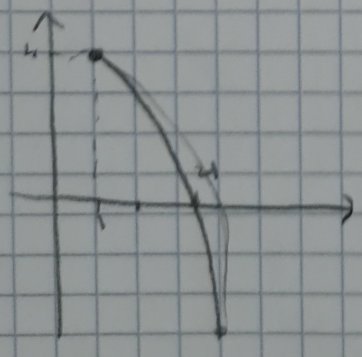


$$\int_0^3 9 - \int_0^3 x^2 = \quad 9x\Big|_0^3 - \frac{x}{3}\Big|_0^3 = 21$$

$$f'(x) = (5 - \sqrt{x^3})' \qquad x^{\frac{3}{2}}$$

$$= 0 - \frac{3}{2} x^{\frac{1}{2}}$$

48-



$$L = \int_0^4 \sqrt{1 + \left[-\frac{3}{2}\sqrt{x}\right]^2}\, dx$$

$$= \int_0^4 \sqrt{1 + \frac{9}{4}x} \quad + \quad \int_{10}^4 \sqrt{\frac{4+9x}{4}}$$

13-

i) $\int \frac{1}{x}(1+\ln^2 x)\,dx$

$= \int \frac{1}{x}\,dx + \int \frac{\ln^2 x}{x}\,dx$

$= \ln|x| + c + \int \frac{1}{x} \cdot \ln x^2$

$\rightarrow \int f'(x)g(x) = f(x)g(x) - \int f(x)g'(x)$

j) $\int \frac{2+\sqrt{\operatorname{arctg}(2x)}}{1+4x^2}\,dx = \int \frac{2}{1+4x^2}\,dx + \int \frac{\sqrt{\operatorname{arctg}(2x)}}{1+4x^2}\,dx$

$= \frac{1}{2} \cdot \int \frac{8}{1+4x^2}\,dx + \int \frac{1}{1+4x^2} \cdot (\operatorname{arctg}(2x))^{1/2}\,dx$

$\int \frac{u'}{u}\,dx = \ln|u| + c$

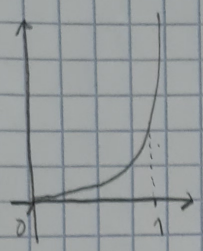
$\int u' u^\alpha = \frac{u^{\alpha+1}}{\alpha+1} + c$

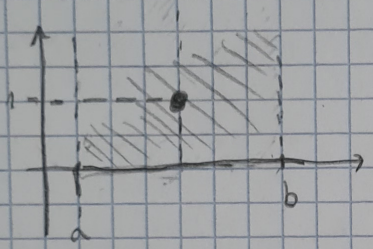
$= \frac{1}{4} \cdot \ln|1+4x^2| + \frac{1}{2} \int \frac{2}{1+4x^2} \cdot (\operatorname{arctg}(2x))^{1/2}$

$= \frac{1}{4}\ln|1+4x^2| + \frac{1}{2}\left(\frac{(\operatorname{arctg}(2x))^{3/2}}{3/2}\right) + c, \quad c \in \mathbb{R}$

$= \frac{1}{4}\ln|1+4x^2| + \frac{(\operatorname{arctg}(2x))^{3/2}}{3} + c, \quad c \in \mathbb{R}$

16- $\int_0^1 x^3 \, dx$

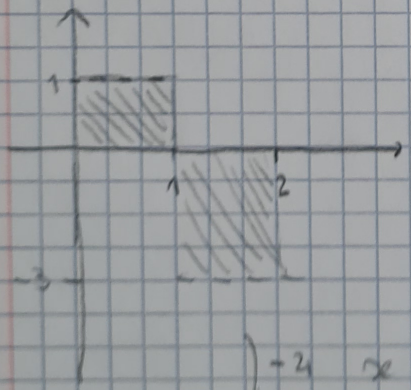




$\int_a^b f(x) \, dx \geq 0$

$\forall x \in [a,b], f(x) \neq 0$

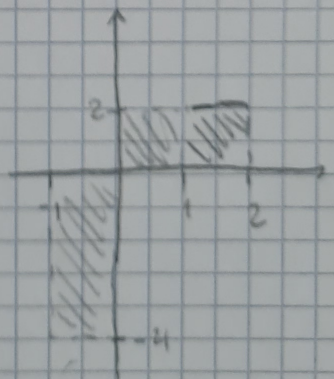
26- $S_n(x) = \begin{cases} 1 \text{ si } x \in [0,1] \\ -3 \text{ si } x \in]1,2] \end{cases}$



$A = \int_0^1 1 + \left(-\int_1^2 -3\right)$

$= x\Big|_0^1 + \left(-(-3x)\Big|_1^2\right) = 1 - 3 = 4$

ii) $g(x) = \begin{cases} -4 & \text{si } x \in [-1,0[\\ 2 & \text{si } x \in [0,1] \\ 2 & \text{si } x \in]1,2] \end{cases}$



$A = -\left(\int_{-1}^{0} -4\right) + \int_0^1 2 + \int_1^2 2 = -\left(-4x\Big|_{-1}^{0}\right) + 2x\Big|_0^1 + 2x\Big|_1^2$

$= 4 + 2 + 2 = 8$