→ Estatística aplicada - Ficha 3

1- $f(x) = \frac{|x-2|}{7}$ $\quad x = -1, 0, 1, 3$ $\qquad E[X] = \sum_{x=i}^{m} x_i \cdot f(x)$

$E[X] = -1 \times \frac{3}{7} + 0 \times \frac{2}{7} + 1 \times \frac{1}{7} + 3 \times \frac{1}{7} = \frac{1}{7}$

• média $= \frac{\frac{3}{7} + \frac{2}{7} + \frac{1}{7} + \frac{1}{7}}{4} = \frac{1}{4}$

$Var[X] = \left(-1-\frac{1}{4}\right)^2 \times \frac{3}{7} + \left(0-\frac{1}{4}\right)^2 \times \frac{2}{7} + \left(1-\frac{1}{4}\right)^2 \times \frac{1}{7} + \left(3-\frac{1}{4}\right)^2 \times \frac{1}{7} =$

$= 0{,}6694 + \frac{1}{56} + \frac{9}{112} + \frac{121}{112} = 1{,}8$

→ Variavel a. discreta

$E[X^2] = \sum_i x_i^2 \, P(X = x_i)$

→ Variavel a. continua

$E[X^2] = \int_{-\infty}^{\infty} x^2 f(x)\, dx$

2- $f(x) = \frac{1}{8}(x+1) \quad 2 \leq x \leq 4$

• $E[X]$ < V. a continua / V. a discreta

• $V[X]$ < V. a continua / V. a discreta

• E

$E[$ … $+ x = \frac{1}{8}\left[\frac{x^3}{3} + \frac{x^2}{2}\right]_2^4 =$

$\frac{1}{8}\left(\frac{\ }{3} \quad \frac{\ }{2} \quad \frac{\ }{3} \right) \ldots \left(\frac{4}{3}\right) = \frac{37}{12} \approx 3{,}083$

• $Var[X] = E[X^2] - (E[X])^2$

• $E[X^2] = \int_2^4 x^2 \cdot \frac{1}{8}(x+1)\, dx = \frac{1}{8}\int_2^4 x^3 + x^2 \, dx = \frac{1}{8}\left[\frac{x^4}{4} + \frac{x^3}{3}\right]_2^4 =$

$= \frac{1}{8}\left(\frac{4^4}{4} + \frac{4^3}{3} - \left(\frac{2^4}{4} + \frac{2^3}{3}\right)\right) = \frac{1}{8}\left(\frac{236}{3}\right) = \frac{59}{6} \approx 9{,}83$

$Var[X] = 9{,}83 - (3{,}08)^2 = 0{,}3436$

3 -

$$f(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 2-x & 1 \leq x < 2 \\ 0 & \text{outros valores} \end{cases}$$

$$\bullet\ E[X] = \int_0^1 x \times x + \int_1^2 x \times (2-x)$$

$$= \int_0^1 x^2 + \int_1^2 2x - x^2 = \left[\frac{x^3}{3}\right]_0^1 + \left[x^2 - \frac{x^3}{3}\right]_1^2$$

$$= \frac{1}{3} + \Big(\underbrace{\left(4 - \frac{8}{3}\right)}_{\frac{4}{3}} - \underbrace{\left(1 - \frac{1}{3}\right)}_{\frac{2}{3}}\Big) = 1$$

$$\bullet\ E[X^2] = \int_0^1 x^2 \times x + \int_1^2 x^2 \times (2-x)$$

$$= \int_0^1 x^3 + \int_1^2 2x^2 - x^3 = \left[\frac{x^4}{4}\right]_0^1 + \left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_1^2$$

$$= \frac{1}{4} + \Big(\underbrace{\left(\frac{16}{3} - \frac{16}{4}\right)}_{\frac{4}{3}} - \underbrace{\left(\frac{2}{3} - \frac{1}{4}\right)}_{\frac{5}{12}}\Big) = \frac{7}{6}$$

$$Var[X] = \frac{7}{6} - 1 = \frac{1}{6} \approx 0{,}167$$

4 -

$$f(x) = \frac{x}{15} \qquad x = 1,2,3,4,5$$

a)

$$E[X] = 1 \times \frac{1}{15} + 2 \times \frac{2}{15} + 3 \times \frac{3}{15} + 4 \times \frac{4}{15} + 5 \times \frac{5}{15} = 3{,}67$$

$$E[X^2] = 1^2 \times \frac{1}{15} + 2^2 \times \frac{2}{15} + 3^2 \times \frac{3}{15} + 4^2 \times \frac{4}{15} + 5^2 \times \frac{5}{15} = 15$$

$$Var[X] = E[X^2] - (E[X])^2 = 15 - (3{,}67)^2 = 1{,}5311$$

4-

b)

$$E[(3X+2)^2] = (3\times1+2)^2\times\frac{1}{15} + (3\times2+2)^2\times\frac{2}{15} + (3\times3+2)^2\times\frac{3}{15} +$$

?

$$+ (3\times4+2)^2\times\frac{4}{15} + (3\times5+2)^2\times\frac{5}{15}$$

$$= \frac{25}{15} + \frac{128}{15} + \frac{121}{5} + \frac{784}{15} + \frac{289}{3}$$

$= 183$

- $E[aX+b] = aE[X] + b$
- $E[X+Y] = E[X] + E[Y]$
- $E[aX] = aE[X]$
- $E[a] = a$
- $E[X-Y] = E[X] - E[Y]$

5-

$$f(x) = \begin{cases} \frac{1}{x(\ln 3)} & 1 \leq x \leq 3 \\ 0 & \text{outros valores} \end{cases}$$

a)

$$E[X] = \int_1^3 x \times \frac{1}{x(\ln 3)} = \frac{1}{\ln 3}\int_1^3 1 = \frac{1}{\ln 3}(3-1) = \frac{2}{\ln 3} = 1,8205$$

$$E[X^2] = \int_1^3 x^2 \frac{1}{x(\ln 3)} = \frac{1}{\ln 3}\int_1^3 x = \frac{1}{\ln 3}\left[\frac{x^2}{2}\right]_1^3 =$$

$$= \frac{1}{\ln 3}(4) = 3,6410$$

$$E[X^3] = \int_1^3 x^3 \frac{1}{x\ln 3} = \frac{1}{\ln 3}\int_1^3 x^2 = \frac{1}{\ln 3}\left[\frac{x^3}{3}\right]_1^3 =$$

$$= \frac{1}{\ln 3}\left(\frac{26}{3}\right) \approx 7,8887$$

$$V[X] = E[X^2] - (E[X])^2 = 3,6410 - (1,8205)^2 = 0,3268$$

b)

$$E[X^3+2X^2-3X+1] = E[X^3]+2E[X^2]-3E[X]+1$$

$$= 7{,}8887 + 2(3{,}6410) - 3(1{,}8205) + 1$$

$$= 10{,}7092$$