→ Folha suplementar 2

1-

X: v.a representa o nº de embalagens de um medicamento vendidos na farmácia

$$X: \begin{cases} 0 & 1 & 2 & 3 & 4 & 5 \\ 0,05 & a & 0,2 & 0,15 & 0,3 & 2a \end{cases}$$

• O espaço amostral $\Omega = \{0,1,2,3,4,5\}$

$P(\Omega) = 1$ logo $P(0)+P(1)+P(2)+P(3)+P(4)+P(5)=1$

$0,05 + a + 0,2 + 0,15 + 0,3 + 2a = 1$

$3a = 1 - 0,70 \Leftrightarrow 3a = 0,30 \Leftrightarrow a = 0,10$ c.q.m

$$F_x = \begin{cases} 0 & x < 0 \\ 0,05 & \text{se } 0 \leq x < 1 \\ 0,05 + a & \text{se } 1 \leq x < 2 \\ 0,25 + a & \text{se } 2 \leq x < 3 \\ 0,40 + a & \text{se } 3 \leq x < 4 \\ 0,70 + a & \text{se } 4 \leq x < 5 \\ 0,70 + 3a & \text{se } 5 \leq x \end{cases}$$

b)

i) $P(X \geq 3) = 1 - P(X < 3) = 1 - 0,35 = 0,65$

ii) $P(X > 3) = 1 - P(X \leq 3) = 1 - 0,5 = 0,5$

iii) $P(X \leq 3) = 0,5$

c)

d)

i) $P(X \leq 2 \mid X \leq 4) = \dfrac{P((X \leq 2) \cap (X \leq 4))}{P(X \leq 4)} = \dfrac{P(X \leq 2)}{P(X \leq 4)} = \dfrac{0,35}{0,8}$

ii) $P(X > 2 \mid X \leq 4) = \dfrac{P(X > 2 \cap X \leq 4)}{P(X \leq 4)} = \dfrac{P(X = 3) + P(X = 4)}{P(X \leq 4)} = \dfrac{0,15 + 0,3}{0,8}$

$= 0,45$

probabilidade de se terem vendido mais que 2 e menos ou 4 embalagens ou seja terem sido vendidos 3 ou 4 embalagens

iii) $\frac{P(X=4 \mid X \geq 4)}{P(X \geq 4)} = \frac{P(X=4)}{P(X \geq 4)} = \frac{0,3}{0,8}$

2-

a) $\int_{-\infty}^{+\infty} f(x) = 1 \Rightarrow \int_{-\infty}^{1} 0\,dx + \int_{1}^{3} kx\,dx + \int_{3}^{+\infty} 0\,dx = 1$

$\Rightarrow 0 + k \int_{1}^{3} \frac{x^2}{2} dx + 0 = 1$

$\Rightarrow k \times \left(\frac{x^2}{2} \Big|_1^3 \right) = 1 \Rightarrow k \times \left(\frac{9}{2} - \frac{1}{2} \right) = 1 \Leftrightarrow k \times 4 = 1$

$\Rightarrow k = \frac{1}{4}$

b)

i) "mais que 1,30h"

$\frac{1}{4} \times \frac{x^2}{2} \Big|_1^c = \frac{1}{8} \times x^2 \Big|_1^c = \frac{1}{8}(c^2 - 1)$

$$F(c) = \begin{cases} \int_{-\infty}^{1} 0\,dx & x \quad c \leq 1 \\ \int_{-\infty}^{1} 0\,dx + \int_{1}^{c} \frac{1}{4} x\,dx \\ \int_{-\infty}^{1} 0\,dx \quad \int_{1}^{3} \frac{1}{4} x + \int_{3}^{+\infty} 0\,dx \end{cases} = \begin{cases} 0 & x \quad c \leq 1 \\ \frac{1}{8}(c^2 - 1) & x \quad 1 \leq c \leq 3 \\ 1 & x \quad 3 \leq x \end{cases}$$

$P(X > 1,5) = 1 - P(X \leq 1,5) = 1 - \frac{1}{8}(1,5^2 - 1) = \frac{27}{32}$

ii) "pelo menos 1.30h"

1,

$P(X \geq 1,5) = 1 - P(X \leq 1,5) = \frac{27}{32}$

1,15

iii) "pelo menos 1:15 h e no máximo 2:00h"

$P(X \geq 1,15 \cap P \leq 2) = P(1,25 \leq X \leq 2) = P(1,25 \leq X \leq 2) = F(2) - F(1,25)$

$= \left(\frac{1}{8}(4-1)\right) - \left(\frac{1}{8}(1,25^2 - 1)\right)$

$= \frac{39}{128}$

c) $\frac{P(X \geq 2 \mid X \geq 1,5)}{P(} = \frac{P(X \geq 2 \mid X \geq 1,5)}{P(X \geq 1,5)} =$

$= \frac{P(X \geq 2)}{P(X \geq 1,5)} = \frac{1 - P(X \leq 2)}{P(X \geq 1,5)} = \frac{1 - \frac{1}{8}(4-1)}{\frac{1}{8}(1,5^2 - 1)} = \frac{20}{27}$