→ Folha suplementar 3

2-

função densidade de prob. é dada por: $f(x) = \begin{cases} 12x^2(1-x) & \text{se } 0 < x < 1 \\ 0 & \text{se c.c} \end{cases}$

• função distribuição:

$$F_X(e) = \begin{cases} \int_{-\infty}^{0} 0 & \text{se } x < 0 \\ \int_{-\infty}^{0} 0 dx \int_0^x 12x^2(1-x) & \text{se } 0 < x < 1 \\ \int_{-\infty}^{0} 0 dx + \int_0^1 12x^2(1-x) + \int_1^{+\infty} 0 dx & \text{se } 1 < x \end{cases}$$

$\int 12x^2 + \int 12x^3$

$= 12 \times \frac{x^3}{3} + 12\frac{x^4}{4}$

$4x^3 + 3x^4$

$$\Rightarrow \begin{cases} 0 & \\ 0 + \int_0^e 12x^2 - 12x^3 & \text{se } 0 < x < 1 \\ 0 + \int_0^1 12x^2 - 12x^3 + 0 & \text{se } 1 < x \end{cases} \Rightarrow \begin{cases} 0 & \text{se } x < 0 \\ 4x^3 - 3x^4 \Big|_0^e & \text{se } 0 < x < 1 \\ 4x^3 - 3x^4 \Big|_0^1 & \text{se } 1 < x \end{cases}$$

$$\Rightarrow \begin{cases} 0 & \text{se } e < 0 \\ 4e^3 - 3e^4 & \text{se } 0 < e < 1 \\ 1 & \text{se } 1 < e \end{cases}$$

utiliza-se a função densidade ⚠

$$E[X] = \int_{-\infty}^{0} 0\, dx + \int_0^1 12x^2(1-x)x \int_1^{+\infty} 0 \times x\, dx =$$

$$= 0 + \int_0^1 12x^3 - 12x^4 + 0 = 12\frac{x^4}{4} - \frac{12x^5}{5}\Big|_0^1 =$$

$$= 3 - \frac{12}{5} - (0) = \frac{15}{5} - \frac{12}{5} = \frac{3}{5}$$

~~→ porque tem de ser positivo~~

?

$$Var[X] = \int_{-\infty}^{+\infty} (x - E[X])^2 f(x) dx$$

$$= \int_{-\infty}^{0} (x - 3/5) \times 0\, dx + \int_0^1 (x - \tfrac{3}{5}) \times (12x^2(1-x))\, x\, dx + \int_1^{+\infty} (x - \tfrac{3}{5}) \times 0\, dx$$

$\frac{36x^2}{5} = \frac{36x^3}{\frac{36}{15}}$

$$= \int_0^1 (x - 3/5)(12x^2 - 12x^3)\, x\, dx$$

$\int 12x^3 - \frac{12x^4}{4}$

$12x^4 = \frac{12x^5}{5}$

$$= \int_0^1 \left(12x^3 - 12x^4 - \frac{36}{5}x^2 + \frac{36}{5}x^3\right) x\, dx = 3x^4 - \frac{12x^5}{5} - \frac{36}{15}x^3 + \frac{36}{20}x^4$$

$=$

b) $\begin{cases} 10 €/l & alcool < 1/3 \\ 15 €/l & 1/3 \leq alcool \leq 2/3 \\ 20 €/l & 2/3 \leq alcool \end{cases}$

	8	13	18
fmp			

c) 5 amostras aleatória

Y: v.a que representa o nº de unidades cuja percentagem de alcool é inferior a 1/3

$Y \sim Bin(5, \frac{1}{9})$

$Y = \begin{cases} 0 & 1 & 2 & 3 & 4 & 5 \\ \left(\frac{8}{9}\right)^5 & \frac{8^4}{9^5} & 10 \times \frac{8^3}{9^5} & 10 \times \frac{8^2}{9^5} & 5 \times \frac{8}{9^5} & \left(\frac{1}{9}\right)^5 \end{cases}$

$P(Y=0) = \binom{5}{0} \times \left(\frac{1}{9}\right)^0 \times \left(\frac{8}{9}\right)^5 = \left(\frac{8}{9}\right)^5$

$P(Y=1) = \binom{5}{1} \times \left(\frac{1}{9}\right)^1 \times \left(\frac{8}{9}\right)^4 = 5 \times \frac{8^4}{9^5}$

$P(Y=2) = \binom{5}{2} \times \left(\frac{1}{9}\right)^2 \times \left(\frac{8}{9}\right)^3 = 10 \times \frac{8^3}{9^5}$

$P(x=3) = \binom{5}{3} \times \left(\frac{1}{9}\right)^3 \times \left(\frac{8}{9}\right)^2 = 10 \times \frac{8^2}{9^5}$

$P(x=4) = \binom{5}{4} \times \left(\frac{1}{9}\right)^4 \times \left(\frac{8}{9}\right)^1 = 5 \times \frac{8}{9^5}$

$P(x=5) = \binom{5}{5} \times \left(\frac{1}{9}\right)^5 \times \left(\frac{8}{9}\right)^0 = \left(\frac{1}{9}\right)^5$

3-

- 2 extrações sem reposição
- 3 bolas numeradas de 1 a 3

X: V.a que representa o nº da primeira bola extraída

Y: V.a que representa o máximo dos números extraídos

$$X: \left\{ \begin{array}{ccc} 1 & 2 & 3 \\ 1/3 & 1/3 & 1/3 \end{array} \right. \qquad Y: \left\{ \begin{array}{cc} 2 & 3 \\ \frac{1}{3} & \frac{2}{3} \end{array} \right. \rightarrow$$ nunca pode ser 1 porque não há reposição

- $E[X] = 1 \times \frac{1}{3} + 2 \times \frac{1}{3} + 3 \times \frac{1}{3}$

$= 2$

- $E[Y] = 2 \times \frac{1}{3} + 3 \times \frac{2}{3} = \frac{8}{3}$

- $Var[X] = (1-2)^2 \times \frac{1}{3} + (2-2)^2 \times \frac{1}{3} + (3-2)^2 \times \frac{1}{3}$

$= \frac{1}{3} + 0 + \frac{1}{3} = \frac{2}{3}$

- $Var[Y] = \left(2 - \frac{8}{3}\right)^2 \times \frac{1}{3} + \left(3 - \frac{8}{3}\right)^2 \times \frac{2}{3}$

$= \frac{4}{27} + \frac{2}{27} = \frac{6}{27} = \frac{2}{9}$

b) $P(X=1 \mid Y=3) = \frac{P(X=1 \cap Y=3)}{P(Y=3)} = \frac{\frac{1}{2}}{\frac{2}{3}} = \frac{3}{4}$

$P(Y=1 \mid X=3) = 0$